The Rodman's Guide to Madison Vertical Datums

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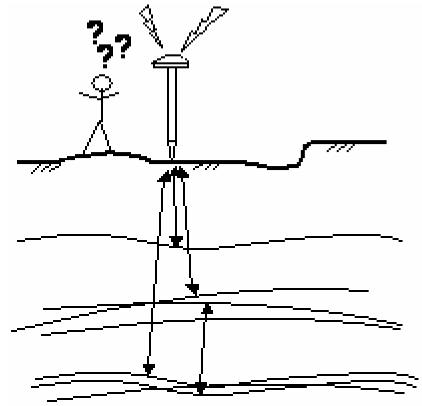

1.1. INTRODUCTION This document explains the various vertical datums used for determining elevations in the City of Madison, Wisconsin area. It is based on the author's experience and is intended for land surveyors and other spatial data professionals. Sections 2.X on the City, NGVD 29 and NAVD 88 datums are basic information for any spatial data user. Sections 3.X and 4.X are more complex information relevant to the Global Positioning System (GPS). Recommended RTK GPS configuration parameters are compiled in a separate document.

1.2. DISCLAIMER The author has compiled this document to increase understanding of Madison-area vertical datums within the land surveying and spatial data community. Users of this document are solely responsible for their own vertical datum-related measurements, computations and related work. The author and the City of Madison make no representation about the accuracy or completeness of this document, and in no event shall the author or the City be liable for any damages whatsoever resulting from its use.

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Data for the GPS sections of the document were originally obtained from National Geodetic Survey data sheets retrieved on July 30, 2007 and December 2, 2007. Periodic checks through Feb 21, 2011 found no changes to those data sheets. NAD 83(2011)(Epoch 2010.00) and NAVD 88(2012) data were obtained initially from Aug 14, 2012 data sheets, but NAVD 88(2012) elevations were not found for non-leveled stations until Jan 11 2014 data sheets (other data used from Aug 14 2012 data sheets had not changed). Station MADISON S GPS (DF9799) was destroyed in 2010 but has 2011/2012 adjustment values, so it remains in the analysis.

2.1. IF LIFE WERE SIMPLE… Historically, the "City" vertical datum was established at the water level of Lake Monona at some historical moment or average. This was determined to be 845.6 feet above the National Geodetic Vertical Datum of 1929 (NGVD 29), which has also been called the "sea level" or "USGS" datum. The National Geodetic Survey (NGS) later readjusted NGVD 29 to create the North American Vertical Datum of 1988, called NAVD 88(1991) here to distinguish it from subsequent adjustments. The NGS determined the NAVD 88(1991) datum to be about 0.2 feet above NGVD 29 in the Madison area (per VERTCON 2.0 software at www.ngs.noaa.gov). Subtracting 0.2 feet from 845.6 feet means the old "City" datum is theoretically 845.4 feet above NAVD 88(1991). In other words, if a point has a "City" elevation = 100.0 feet, it should have a NGVD 29 elevation = (100.0 + 845.6) = 945.6 feet, and a NAVD 88(1991) elevation = (100.0 + 845.4) = 945.4 feet. Figure 01 illustrates these relationships.

2.2 …IT WOULD BE TOO EASY Unfortunately, it's not that simple, and it never has been. Elevations are not physically measured directly from the datum, because the datum is only a conceptual level surface. Even the "City" datum, based on Lake Monona, is not visible today because it is based on some historical water level. Rather, elevation is computed by measuring *change* in elevation from a bench mark, which has a predetermined elevation relative to datum. If the bench mark elevation is wrong, any elevation determined from it is wrong.

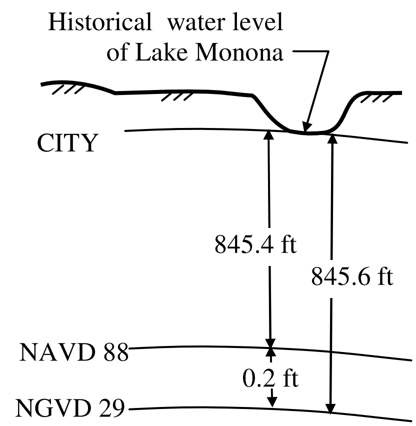


Figure 01. If Life Were Simple (Profile View)

Experience has shown that across the city, the network of bench marks (such as fire hydrant top nuts and section corner monuments) with older "City" elevations is consistent within local areas, but there are "fault lines" between areas. These are most probably due to measurement and computation errors when establishing the bench mark elevations, or subsequent monument disturbance (such as replaced hydrants), rather than post-measurement physical ground shifts. The author has observed inconsistencies of up to 2 feet, and greater ones may exist. Thus, a national bench mark with a NGS-published NAVD 88 elevation of 945.4 feet, and a nearby fire hydrant with a "City" elevation of 100.0 feet, are not necessarily at the same elevation.

Because certain projects such as sewer and road construction usually require very precise elevations, the theoretical datum transformations above cannot be trusted to relate different bench marks in different datums. Bench marks must be checked with current measurements to ensure that any one bench mark hasn't moved, or that a bench mark's published elevation doesn't have measurement or computation errors.

Of course, measurements between bench marks can only check elevation *change* between bench marks. If the measured difference between two bench marks doesn't agree with the difference computed from published elevations, it is impossible to determine, from relative measurements alone, which (if either) of the published bench mark elevations *above datum* is correct. This is why project documentation including bench marks used, published elevation value, source and date of elevation value, and (supposed) datum, are critical to consistent vertical control.

2.3. …AND THEN, THE NAVD 88 READJUSTMENTS In 2007 and again in 2012, NAVD 88 elevations for bench marks (stations) in the Madison area were adjusted by very systematic shifts. NAVD 88(2007) elevations are on average 0.11 feet larger than original NAVD 88(1991) values, and NAVD 88(2012) elevations are on average 0.06 feet larger than NAVD 88(2007) values (Figure 02). Unfortunately, currently (Jan 2014) the NGS refers to all these datums as NAVD 88 without a suffix for the adjustment year, even though the net average shift through 2012 in the Madison area is 0.17 feet. Suffixes have been added in this document to differentiate. Coincidentally, the NAVD 88 datum adjustments through 2012 are heading back toward the NGVD29 datum, so confusing the 4 datums only results in a maximum of 0.2 ft error (assuming no other errors are present).

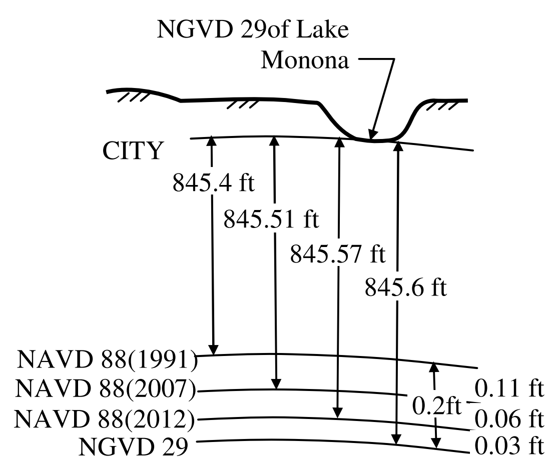


FIGURE 02. The Various NAVD 88 Datums
(Profile view showing average shifts)

The shifts in Figure 02 are average values for the Madison area. They are not necessarily correct for a particular bench mark, which may have measurement errors or may have been disturbed since measurement. Note that as of Sept. 2012, NGS's VERTCON 2.0 software (at www.ngs.noaa.gov/TOOLS) predicts a 0.2 foot shift (with slight variations by location) between NGVD 29 and the *original* NAVD 88(1991), not the later NAVD 88 adjustments.

The NAVD 88 elevations established for section corner monuments in the City of Madison starting in 2004 were in NAVD 88(1991). See http://gis.cityofmadison.com/Madison_PLSS/

2.4. NAVD 88(2007) DETAILS Based on NGS data sheets retrieved July 30, 2007 from www.ngs.noaa.gov, the leveled NAVD 88 elevations of 120 Second Order Class I stations within 25 km (~15 mi) of the Madison GPS base station had NAVD 88 elevations larger than their previous (1991 through 2004) values between +0.04 and +0.15ft, with an average of +0.11ft. The low station at +0.04ft was E 108 (NGS PID# OM0450). It is far west of Madison and an outlier with an older (1991) previous elevation. 89% of the 120 stations had elevation shifts between +0.09 ft and +0.13 ft (within 0.02 ft of the average). A slight tilt is evident between NAVD 88(1991) and NAVD 88(2007), with the larger shifts mostly northerly and westerly and the smaller shifts mostly southerly and easterly. However, these tilts were only hundredths of a foot over ~30 miles. Shifts on Madison's Isthmus were slightly below the average (+0.07ft to +0.08ft).

Figure 03 shows a plot of NAVD 88 elevation changes (1991 to 2007) at these NGS stations. Differential leveling was used to compute NAVD 88(1991) elevations at the old Sayle Street GPS base station (discontinued in 2009) relative to Station 2V02, and at the new Emil Street GPS base station (MAON) relative to Station MADISON S GPS (destroyed in 2010). See Appendix 2 and http://gis.cityofmadison.com/Madison_GPS/

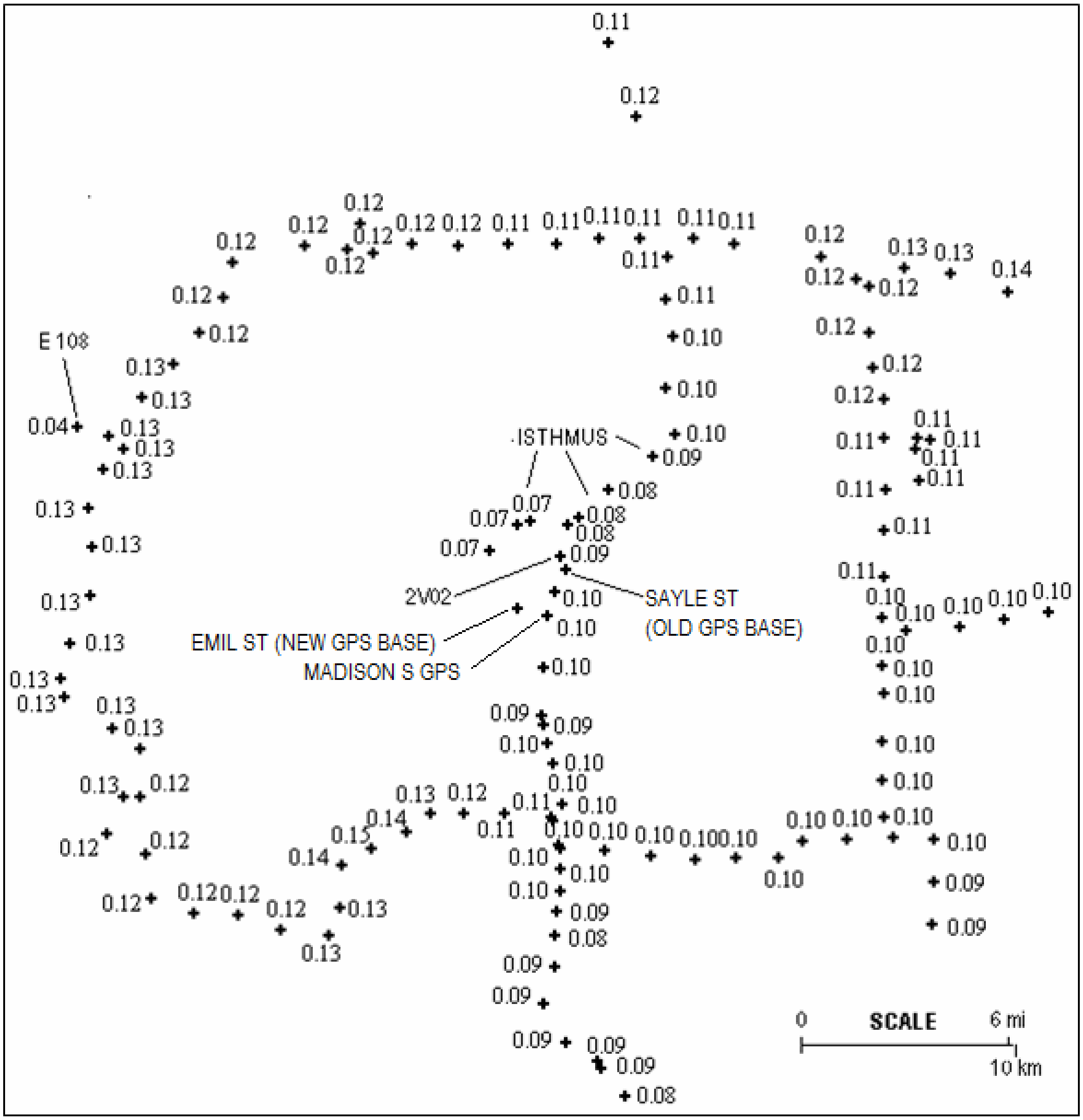


Figure 03: Elevation shift at NGS Bench Marks, NAVD 88(2007) minus NAVD 88(1991), feet

2.5. NAVD 88(2012) DETAILS For the analysis of the May 2012 NAVD 88 adjustment, NGS data sheets were retrieved August 26, 2012 from www.ngs.noaa.gov for stations with leveled NAVD 88 elevations of Second Order Class I or better accuracy (all are Second Order Class 1) within 25 km (~15 mi) of the old Sayle St GPS base station. Of the 159 stations in this selection set, 43 were excluded because most recent NAVD 88 elevation was from June 1991. Station 2V11 (PID# DF9957) was also excluded because of vertical instability noted on the data sheet. 115 were retained for analysis, including MADISON S GPS (PID# DF9799) which was destroyed in 2010 but included in the 2012 adjustment. 112 of the 115 were in the July 2007 comparison of NAVD 88(1991) and NAVD 88(2007).

These 115 stations had NAVD 88(2012) elevations larger than their NAVD 88(2007) values by between +0.04 and +0.08ft, with an average of +0.06 ft. The 53 of the 115 within 17 km (10.6 mi) from the old Sayle St base station near the center of Madison have differences within +0.06 to +0.07ft (average rounds to 0.06 ft). A very slight tilt is evident between NAVD 88(2007) and NAVD 88(2012), with larger shifts to the northeast and smaller shifts to the west. However, these tilts were only hundredths of a foot over ~30 miles.

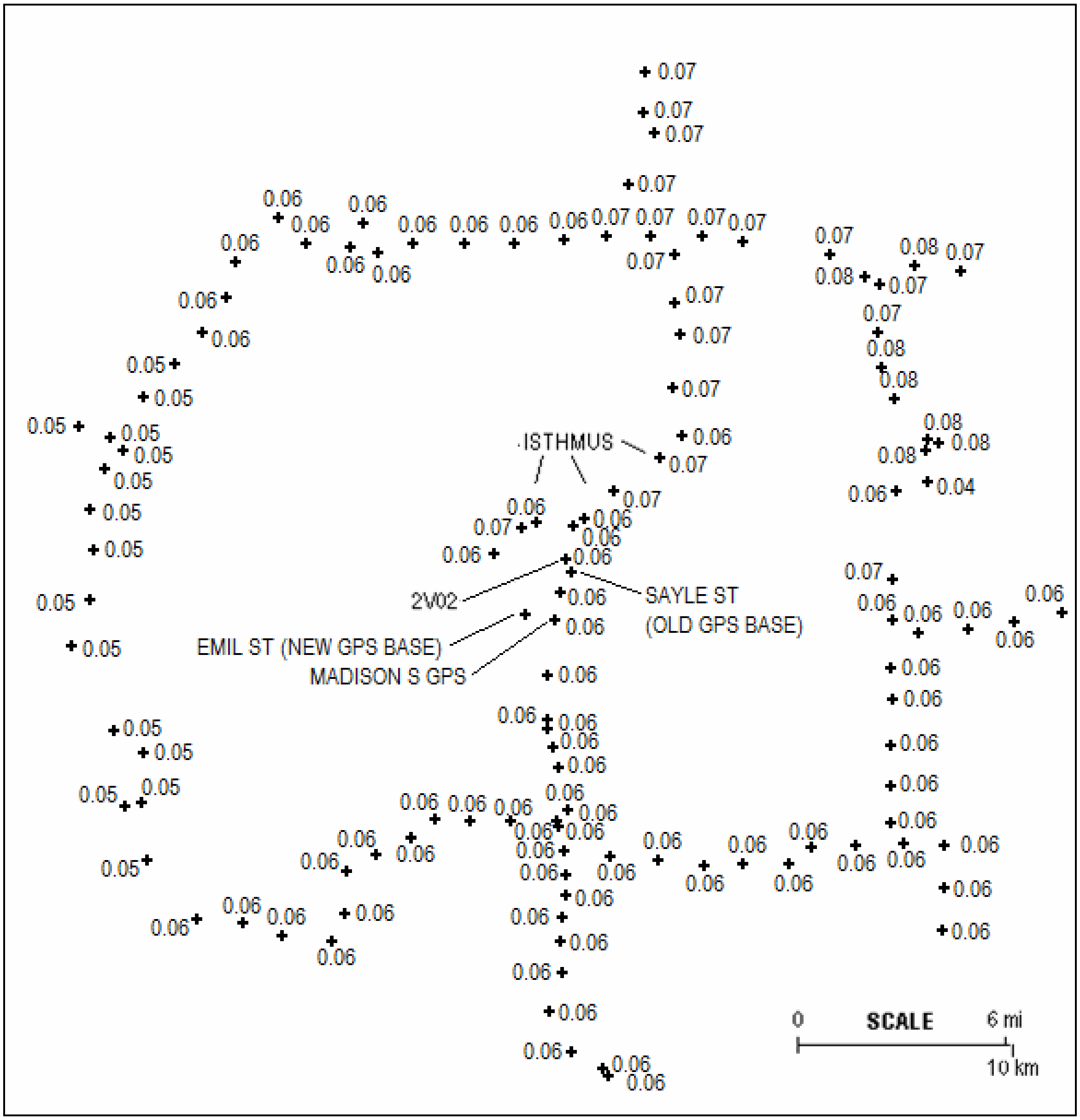


Figure 04: Elevation shift at NGS Bench Marks, NAVD 88(2012) minus NAVD 88(2007), feet

2.6. WHICH NAVD 88 DO I HAVE?

Section Corner Tie Sheets produced for the City of Madison (mostly by Carl Sandsnes) show original NAVD 88 (1991) elevations. As of January 2014, the city plans to stay on the original NAVD 88 datum. http://gis.cityofmadison.com/Madison_PLSS/

NGS Data Sheets do not currently (Jan 2014) differentiate between different adjustments of NAVD 88, but it can usually be determined from the notes. In the Madison area, the author has found that NAVD 88 elevations with dates before April 2007 are NAVD 88(1991), April 2007 or after are NAVD 88(2007), and after May 2012 are NAVD 88(2012). However, it may be difficult to tell. For example, the data sheet for station MADISON GPS (PID# DG4910) retrieved on July 30, 2007 showed a GPS-derived NAVD 88 elevation of 272.26 m (893.24 ft, unrounded). The data sheet for the same station retrieved on August 28, 2007 showed the same elevation as 272.30 m (893.37 ft, unrounded), and on January 11, 2014 as 272.31m (893.40 ft, unrounded). The previous values appear under the Superceded Survey Control section of the January 11, 2014 data sheet, but not on the August 28, 2007 data sheet. Information in the data sheet about dates the station was visited may be useful.

Another important note is that as of January 2014, NGS's VERTCON 2.0 software (at www.ngs.noaa.gov/TOOLS) predicts a 0.2 foot shift (with slight variations by location) in the Madison area between NGVD 29 and NAVD 88(1991), not the later NAVD 88 adjustments. VERTCON may be updated in the future.

3.1. GPS: ELLIPSOID VS. GEOID The vertical datums listed above (City, NGVD 29, various NAVD 88) are all based on a level surface called the geoid. Without getting into too much detail, the geoid is basically defined by gravity. The geoid surface is everywhere at right angles to gravity, such that a marble placed on it would not roll anywhere. While the geoid is basically flat over a small area, it is not a regular surface, basically because the Earth's mass is not uniform, and therefore the force of gravity isn't either. Traditional surveying instruments such as levels and total stations measure elevation changes with respect to the direction of gravity, so they produce data consistent with these geoid-based datums.

The Global Positioning System (GPS) measures elevation changes with respect to a different type of surface – the ellipsoid. In contrast to the geoid, which is defined by a physical force (gravity), the ellipsoid is defined by a mathematical formula (an ellipse rotated about the Earth's axis). In order to convert GPS-measured ellipsoid heights into elevations above the geoid, the relation between the geoid and ellipsoid must be known.

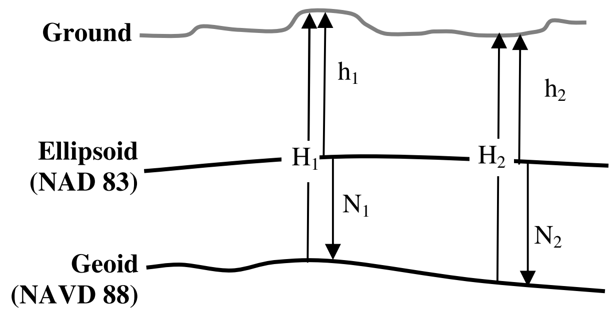


Figure 05: Geoid vs. Ellipsoid
H = elevation above geoid (e.g. NAVD 88)
h = height above ellipsoid (e.g. NAD 83)
N = geoid separation. **N=h–H** (N negative in this area)

Unfortunately, it is not perfectly known, because the geoid and ellipsoid are such different surfaces. The National Geodetic Survey produces "geoid models" which estimate the separation between the geoid and ellipsoid for a given location. GEOID03, GEOID09 and GEOID12A (produced in 2003, 2009 and 2012) are the National Geodetic Survey's most recent (as of 2014) geoid models for this area. They predict geoid separations (N) between ellipsoid heights above the North American Datum of 1983 (NAD 83 - ellipsoid) and elevations above the North American Vertical Datum of 1988 (NAVD 88 – geoid). Details and computations are available at: http://www.ngs.noaa.gov/TOOLS/

Figure 06 shows the geoid separations predicted by GEOID03 in the Madison area as a contour map. GEOID09 and GEOID12A are similar. Note the following from observing the map:

- **The geoid and ellipsoid aren't parallel.** If they were, there would be no contours – just a single elevation difference between the two surfaces. Rather, the geoid is sloped in a generally northeast-southwest direction relative to the ellipsoid, at a rate of about 0.1 feet per mile.
- **The slope isn't constant.** The contours aren't evenly spaced, which means the slope changes. The slope between Verona and Middleton is about 0.07 feet per mile, while the slope between Waunakee and DeForest is about 0.14 feet per mile.
- **The slope changes direction.** The contours are curved, which means the direction of slope changes. Farther east and northwest of Madison, the slope is more north-south.

The relationship is irregular primarily because the geoid surface is defined by gravity, which is irregular. While the rate of change (~0.1 feet per mile) may seem small, GPS is capable of measuring over very long distances, so the cumulative difference can be significant.

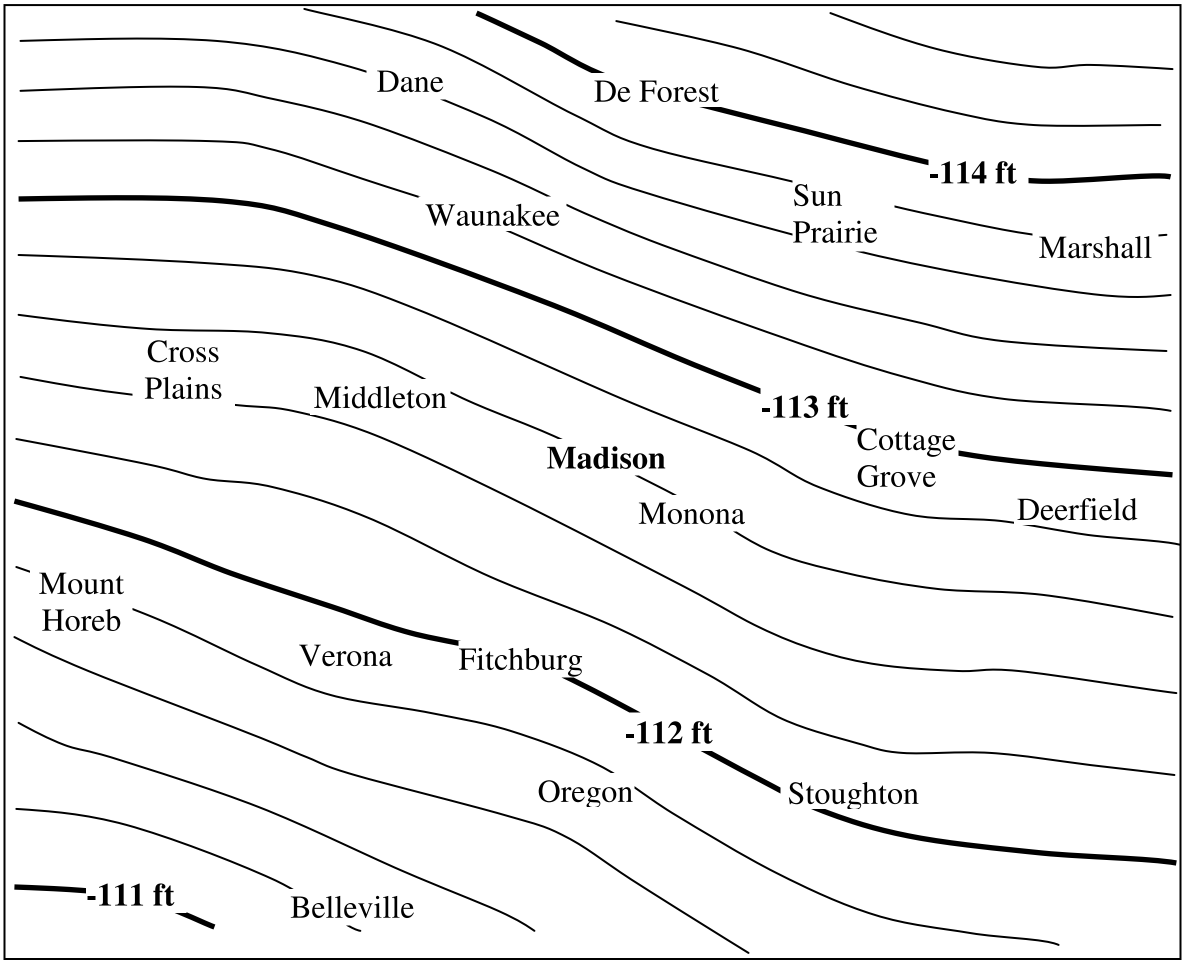


FIGURE 06: Geoid Separations Predicted by GEOID03 in the Madison Area

3.2. ELLIPSOID DATUMS The ellipsoid is a datum for both horizontal (latitude longitude) and vertical (ellipsoid height) positions. While GPS is a three-dimensional positioning system that simultaneously determines all three of these coordinates, the horizontal has traditionally been considered separately from the vertical because vertical accuracy has been more difficult to achieve. More recent datum adjustments have treated the ellipsoid as a single three-dimensional datum. The North American Datum of 1983 (NAD 83) is the most recent definition of the ellipsoid. Its various horizontal and vertical adjustments in the Madison area are summarized in Table 01:

TABLE 01: Recent Ellipsoid Datums in the Madison area

Horizontal Datum Name	Corresponding Vertical Datum Name	Notes for Madison Area (shifts not necessarily applicable elsewhere)
NAD 83(1986)	"NAD 83"	Original adjustment, completed in 1986. Ellipsoid heights not well defined.
NAD 83(1991)	"NAD 83" *(ellipsoid heights dated 1991)*	1991 adjustment, also called High Accuracy Reference Network (HARN) or High Precision Geodetic Network (HPGN). Ellipsoid heights not named "NAD 83(1991)", but rather by date of observation (circa 1991). Approximate horizontal coordinate changes in the Madison area from 1986 to 1991 are approximately N +0.8 ft, E -0.2 ft according to NGS NADCON utility.
NAD 83(1997)	"NAD 83" *(ellipsoid heights dated 1999-2005)*	1997 adjustment. In the Madison area, ellipsoid heights dated 1999-2005 are 0.30 to 0.41 ft smaller than 1991. Ellipsoid heights still not named "NAD 83(1997)", but rather by date of observation. For simplicity in this document, 1999-2005 ellipsoid heights will be called 1997 ellipsoid heights. Average horizontal coordinate changes from 1991 to 1997 in the Madison area are N +0.05 ft, E -0.09 ft.

Horizontal Datum Name	Corresponding Vertical Datum Name	Notes for Madison Area (shifts not necessarily applicable elsewhere)
NAD 83 (CORS96) (EPOCH 2002.0000)	NAD 83 (CORS96) (EPOCH 2002.0000)	Datum for NGS Continuously Operating Reference Stations (CORS) network. Same name for horizontal and vertical datum means ellipsoid is considered a single three-dimensional datum. In the Madison area, horizontal and vertical shift relative to NAD 83(1997) & 1997ellipsoid heights is difficult to distinguish from random measurement error (~ 0.05 ft).
NAD 83(2007) or (NSRS2007)	"NAD 83" *(ellipsoid heights dated 2007)*	The 2007 adjustment used CORS as control, but NGS doesn't call 2007 adjustment "identical" to CORS datum for various reasons, including lack of vertical ground movement modeling. In the Madison area 2007 Ellipsoid heights are about the same as 1997, mostly within 0.03 ft (Table 03 / Figure 08). Average horizontal coordinate changes from 1997 to 2007 in the Madison area are N +0.02 ft, E +0.03 ft.
NAD 83(2011) (EPOCH 2010.00)	NAD 83(2011) (EPOCH 2010.00)	2011(Epoch 2010.00) ellipsoid heights are 0.08 to 0.12 ft (average 0.10 ft) smaller than 2007 values in the Madison area. Note that NAVD 88(2012) elevations are an average of 0.06 ft cm larger than 2007 values, a shift in the other direction) Average horizontal coordinate changes from 2007 to 2011(Epoch 2010.00) are N 0.00 ft, E +0.07 ft in the Madison area. In addition to the adjustment year (2011), the epoch date (2010.00 = start of 2010) is now essential since it has been determined that the ground is moving relative to the datum. According to NGS HTDP 3.2.3 software (www.ngs.noaa.gov/TOOLS), in the Madison area station latitudes (northings) are decreasing about 0.002 ft/yr, west longitudes are decreasing (eastings are increasing) about 0.007 ft/yr, and ellipsoid heights are decreasing about 0.004 ft/yr relative to the NAD 83(2011) ellipsoid.

Although this document is about vertical datums, horizontal NAD 83 coordinate changes since the 1991 adjustment are shown in Table 02 and Figure 07 for reference. These changes have no significant effect on vertical computations, such as getting geoid separations from a geoid model or determining the origin point for geoid shift & tilt parameters (e.g. Table 08 & Table 09). Average changes were computed from the 34 Madison-area NGS stations used for the geoid model analysis (e.g. Table 04). To illustrate how the ground is moving relative to NAD 83(2011), the predicted shift at Epoch 2020.00 (10 years after Epoch 2010.00) is shown, based on -0.002ft/yr north and +0.007ft/yr east.

Table 02: Madison-Area Average Horizontal Coordinate Changes (feet), NAD 83(1991) through NAD83(2011)(Epoch 2010.00), with predicted shift through NAD 83(2011)(Epoch 2020.00).

NAD 83 year	Incr. dN (ft)	Incr. dE (ft)	Cum. dN (ft)	Cum dE (ft)
1991			0.00	0.00
	+0.05	-0.09		
1997			+0.05	-0.09
	+0.02	+0.03		
2007			+0.07	-0.07
	+0.00	+0.07		
2011 (Epoch 2010.00)			+0.07	0.00
	-0.02	*+0.07*		
2011 (Epoch 2020.00)			*+0.05*	*+0.14*

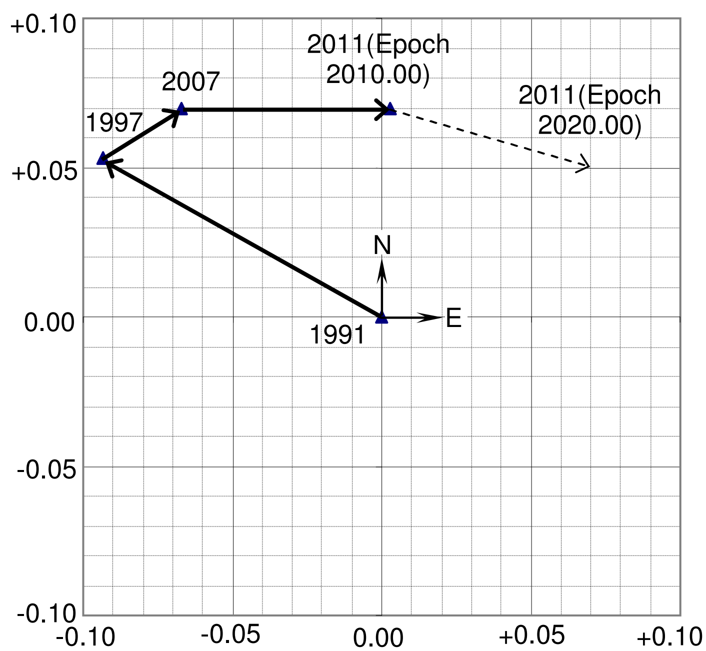


Figure 07: Madison-Area Average Horizontal Coordinate Changes (feet), NAD 83(1991) through NAD83(2011)(Epoch 2010.00), with predicted shift through NAD 83(2011)(Epoch 2020.00).

Figure 08 shows the most recent change in NAD 83 ellipsoid heights, from 1997 to 2007 to 2011(Epoch 2010.00). Table 03 shows 36 NGS stations within 20 km (~12.5 mi) of the Madison GPS base station, with 1997 ellipsoid heights of Fourth Order or Third Order. 35 of the 36 stations had 2007 minus 1997 shifts between -0.04ft and +0.03ft, averaging -0.01ft. The 36th station, ARP 2 MSN (PID #OM1387), is 10 km (~6 mi) north of Madison and had a 1997 to 2007 shift of -0.06ft. It is possible that ARP 2 MSN physically sank between 1997 and 2007, if its 2007 ellipsoid height is in fact computed from new measurements. The NGS data sheet for ARP 2 MSN shows it as "stability type C", which is "of type commonly subject to surface motion." From 1997 to 2007 there appears to be some systematic tilt between the two ellipsoids, with negative shifts to the southwest and positive shifts to the northeast (unrelated to the NAVD 88 tilt). However, a tilt of 0.07 ft over 25 miles is only about 1 part per million, which is undetectable by all but the most precise GPS techniques.

Ellipsoid height changes from 2007 to 2011(Epoch 2010.00) are more consistent, between -0.08 and -0.12 feet, and virtually no slope is visually evident in the spatial distribution.

However, the ultimate goal is geoid-based NAVD 88 elevations, so the conversion from ellipsoid to geoid must be further examined.

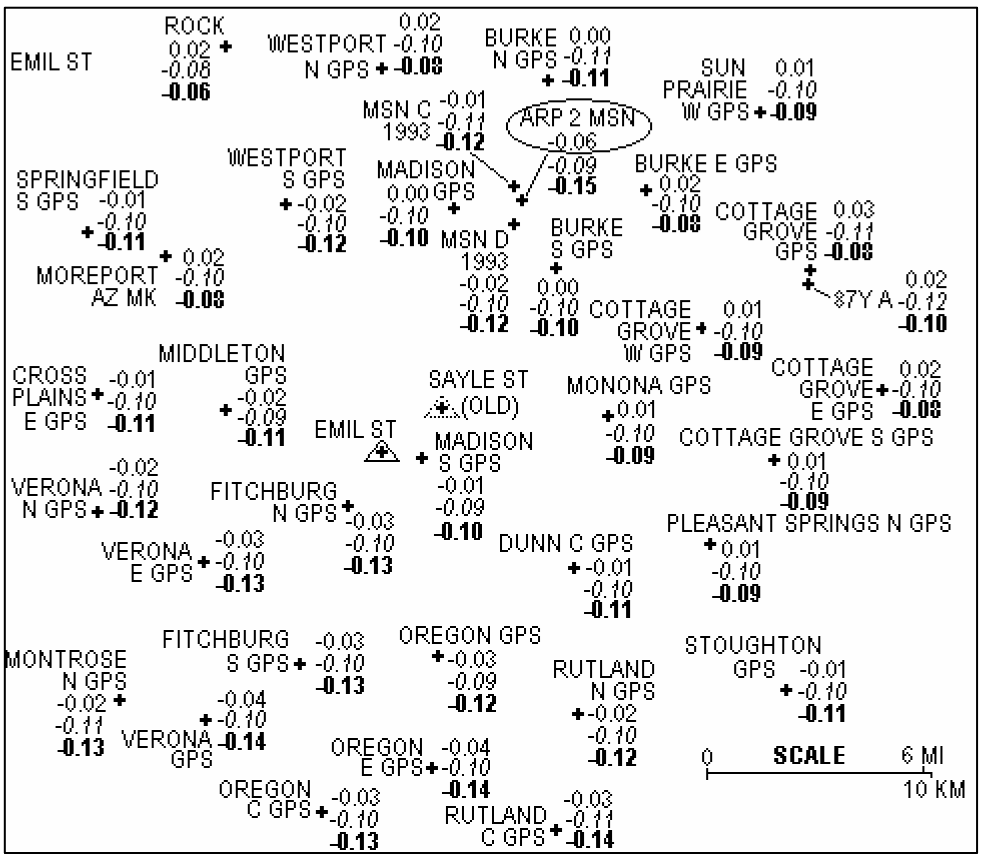


Figure 08: Recent NAD 83 Ellipsoid Height Shifts, feet
regular=2007 minus 1997,
italic=2011(Epoch 2010.00) minus 2007,
bold=2011(Epoch 2010.00) minus 1997

Table 03: NAD 83 Ellipsoid Height Changes 1997 - 2007 – 2011(Epoch 2010.00)
(from NGS Data Sheets retrieved July 31, 2007 and Aug 14 2012)

Station	“1997” Ellip Ht, ft	“1997” Date	2007 Ellip Ht, ft	2011(Epoch 2010.00) Ellip Ht, ft	2007 minus 1997, ft	2011 minus 2007, ft	2011(Epoch 2010.00) minus 1997, ft
MADISON S GPS	755.67	4/2/2004	755.66	755.57	-0.01	-0.09	-0.10
FITCHBURG N GPS	824.23	4/2/2004	824.20	824.10	-0.03	-0.10	-0.13
MONONA GPS	750.52	4/2/2004	750.53	750.43	0.01	-0.10	-0.09
BURKE S GPS	738.42	4/2/2004	738.42	738.32	0.00	-0.10	-0.10
MSN D 1993	743.59	5/3/1999	743.57	743.47	-0.02	-0.10	-0.12
MADISON GPS	780.42	4/2/2004	780.42	780.32	0.00	-0.10	-0.10
DUNN C GPS	746.01	4/2/2004	746.00	745.90	-0.01	-0.10	-0.11
MIDDLETON GPS	967.77	4/2/2004	967.75	967.66	-0.02	-0.09	-0.11
ARP 2 MSN	742.38	4/28/1999	742.32	742.23	**-0.06**	-0.09	**-0.15**
MSN C 1993	743.30	5/3/1999	743.29	743.18	-0.01	-0.11	-0.12
OREGON GPS	866.01	4/2/2004	865.98	865.89	-0.03	-0.09	-0.12
WESTPORT S GPS	759.74	4/2/2004	759.72	759.62	-0.02	-0.10	-0.12
COTTAGE GROVE W GPS	778.98	4/2/2004	778.99	778.89	0.01	-0.10	-0.09
VERONA E GPS	841.50	4/2/2004	841.47	841.37	-0.03	-0.10	-0.13
FITCHBURG S GPS	947.97	4/2/2004	947.94	947.84	-0.03	-0.10	-0.13
BURKE E GPS	812.26	4/2/2004	812.28	812.18	0.02	-0.10	-0.08
PLEASANT SPRINGS N GPS	748.84	4/2/2004	748.85	748.75	0.01	-0.10	-0.09
MOREPORT AZ MK	813.51	9/30/2002	813.53	813.43	0.02	-0.10	-0.08
RUTLAND N GPS	808.44	4/2/2004	808.42	808.32	-0.02	-0.10	-0.12
COTTAGE GROVE S GPS	818.13	4/2/2004	818.14	818.04	0.01	-0.10	-0.09
CROSS PLAINS E GPS	965.35	4/2/2004	965.34	965.24	-0.01	-0.10	-0.11
BURKE N GPS	770.65	4/2/2004	770.65	770.54	0.00	-0.11	-0.11
WESTPORT N GPS	824.63	4/2/2004	824.65	824.55	0.02	-0.10	-0.08
OREGON E GPS	922.54	4/2/2004	922.50	922.40	-0.04	-0.10	-0.14
VERONA N GPS	895.23	4/2/2004	895.21	895.11	-0.02	-0.10	-0.12
87Y A	787.90	4/28/1999	787.92	787.80	0.02	-0.12	-0.10
VERONA GPS	841.22	4/2/2004	841.18	841.08	-0.04	-0.10	-0.14
COTTAGE GROVE GPS	788.30	4/28/1999	788.33	788.22	0.03	-0.11	-0.08
SPRINGFIELD S GPS	945.77	4/2/2004	945.76	945.66	-0.01	-0.10	-0.11
OREGON C GPS	806.73	4/2/2004	806.70	806.60	-0.03	-0.10	-0.13
ROCK	946.44	8/1/2005	946.46	946.38	0.02	-0.08	-0.06
MONTROSE N GPS	815.64	4/2/2004	815.62	815.51	-0.02	-0.11	-0.13
RUTLAND C GPS	803.30	4/2/2004	803.27	803.16	-0.03	-0.11	-0.14
SUN PRAIRIE W GPS	847.52	4/2/2004	847.53	847.43	0.01	-0.10	-0.09
COTTAGE GROVE E GPS	771.30	4/2/2004	771.32	771.22	0.02	-0.10	-0.08
STOUGHTON GPS	776.61	4/2/2004	776.60	776.50	-0.01	-0.10	-0.11

3.3. GEOID RESIDUALS: According to the formula back in Figure 05, the geoid separation (N) is the difference between NAD 83 ellipsoid height (h) and NAVD 88 elevation (H). Since a geoid model (e.g. GEOID03) is only a prediction, the geoid model value (N) will not necessarily match the difference (h-H) computed from published heights. Moreover, there are now multiple adjustments of both NAVD 88 and NAD 83, so it depends on which are used to evaluate the difference.

Figure 09 shows the formula and associated diagram for computing the "geoid residual", which shows how well the geoid model (GEOID03) matches the difference in NAD 83 ellipsoid height (h) and NAVD 88 elevation (H).

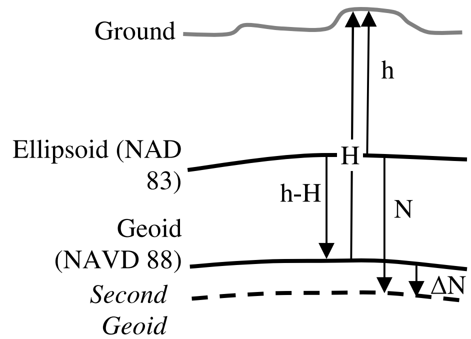


Figure 09: Geoid Residual ΔN = N – (h-H)
H = elevation above geoid (NAVD 88)
h = height above ellipsoid (NAD 83)
N = geoid separation. Ideally, N=h–H. Here, N≠h-H, and (h-N) is shown creating a second geoid.

Table 04 lists 34 NGS stations within 20 km (~12.5 mi) of the Sayle Street base station, having 1997 NAD 83 ellipsoid heights and NAVD 88(1991) elevations. 15 of them have leveled NAVD 88 elevations, and another 19 have GPS-derived NAVD 88 elevations, computed using a "high resolution geoid model" according to NGS data sheets and reported to 1-cm precision.

Note that the 10 "blue book" stations established by the City of Madison (EAGLEWOOD GPS, ELVER PARK GPS, QUARRY COVE GPS, OWEN PARK GPS, EMIL GPS, EDINA-TAYLOR GPS, DOMINION GPS, PATRIOT PARK GPS, EAGLE SCHOOL GPS, and BURR JONES GPS) are not included in this analysis, because their NGS data sheets only report NAVD 88 elevations to the nearest 10 cm.

TABLE 04: GEOID03 Geoid Residuals for NAVD 88(1991) Elevations and 1997 NAD 83 Ellipsoid Heights
From NGS Data Sheets retrieved July 31, 2007

Station	**H** NAVD 88(1991), ft	NAVD 88 source	NAVD 88 Date	**h** 1997 NAD 83 Ellip Ht, ft	Ellip Ht Date	**N** (GEOID03), ft	**h-H**, ft	**ΔN** = N-(h-H), ft
MADISON S GPS	867.91	Leveled	02/25/04	755.67	4/2/04	-112.31	-112.24	-0.07
BURKE S GPS	851.16	Leveled	02/25/04	738.42	4/2/04	-112.86	-112.74	-0.12
OREGON GPS	977.81	Leveled	02/25/04	866.01	4/2/04	-111.86	-111.80	-0.06
FITCHBURG S GPS	1059.61	Leveled	02/25/04	947.97	4/2/04	-111.76	-111.64	-0.12
RUTLAND N GPS	920.33	Leveled	02/25/04	808.44	4/2/04	-111.88	-111.89	0.01
COTTAGE GROVE S GPS	930.73	Leveled	02/25/04	818.13	4/2/04	-112.57	-112.60	0.03
BURKE N GPS	884.02	Leveled	02/25/04	770.65	4/2/04	-113.47	-113.37	-0.10
WESTPORT N GPS	937.82	Leveled	02/25/04	824.63	4/2/04	-113.30	-113.19	-0.11
OREGON E GPS	1034.06	Leveled	02/25/04	922.54	4/2/04	-111.62	-111.52	-0.10
87Y A	900.92	Leveled	02/25/04	787.90	4/28/99	-113.02	-113.02	0.00
VERONA GPS	952.66	Leveled	02/25/04	841.22	4/2/04	-111.61	-111.44	-0.17
COTTAGE GROVE GPS	901.38	Leveled	02/25/04	788.30	4/28/99	-113.06	-113.08	0.02
ROCK	1059.48	Leveled	02/25/04	946.44	8/1/05	-113.20	-113.04	-0.16
SUN PRAIRIE W GPS	961.02	Leveled	02/25/04	847.52	4/2/04	-113.52	-113.50	-0.02
STOUGHTON GPS	888.72	Leveled	02/25/04	776.61	4/2/04	-112.08	-112.11	0.03
FITCHBURG N GPS	936.25	GPS-derived	4/2/04	824.23	4/2/04	-112.16	-112.02	-0.14
MONONA GPS	863.09	GPS-derived	4/2/04	750.52	4/2/04	-112.56	-112.57	0.01
MADISON GPS	893.24	GPS-derived	4/2/04	780.42	4/2/04	-112.93	-112.82	-0.11
DUNN C GPS	858.17	GPS-derived	4/2/04	746.01	4/2/04	-112.21	-112.16	-0.05

Station	**H** NAVD 88(1991), ft	NAVD 88 source	NAVD 88 Date	**h** 1997 NAD 83 Ellip Ht, ft	Ellip Ht Date	**N** (GEOID03), ft	**h-H**, ft	**ΔN** = N-(h-H), ft
MIDDLETON GPS	1079.89	GPS-derived	4/2/04	967.77	4/2/04	-112.25	-112.12	-0.13
ARP 2 MSN	855.38	GPS-derived	4/28/99	742.38	4/28/99	-113.03	-113.00	-0.03
WESTPORT S GPS	872.34	GPS-derived	4/2/04	759.74	4/2/04	-112.78	-112.60	-0.18
COTTAGE GROVE W GPS	891.80	GPS-derived	4/2/04	778.98	4/2/04	-112.83	-112.82	-0.01
VERONA E GPS	953.31	GPS-derived	4/2/04	841.50	4/2/04	-111.96	-111.81	-0.15
BURKE E GPS	925.39	GPS-derived	4/2/04	812.26	4/2/04	-113.19	-113.13	-0.06
PLEASANT SPRINGS N GPS	861.22	GPS-derived	4/2/04	748.84	4/2/04	-112.37	-112.38	0.01
MOREPORT AZ MK	925.88	GPS-derived	9/30/02	813.51	9/30/02	-112.54	-112.37	-0.17
CROSS PLAINS E GPS	1077.39	GPS-derived	4/2/04	965.35	4/2/04	-112.21	-112.04	-0.17
VERONA N GPS	1007.05	GPS-derived	4/2/04	895.23	4/2/04	-112.00	-111.82	-0.18
SPRINGFIELD S GPS	1058.13	GPS-derived	4/2/04	945.77	4/2/04	-112.56	-112.36	-0.20
OREGON C GPS	918.08	GPS-derived	4/2/04	806.73	4/2/04	-111.48	-111.35	-0.13
MONTROSE N GPS	927.07	GPS-derived	4/2/04	815.64	4/2/04	-111.60	-111.43	-0.17
RUTLAND C GPS	914.89	GPS-derived	4/2/04	803.30	4/2/04	-111.63	-111.59	-0.04
COTTAGE GROVE E GPS	884.09	GPS-derived	4/2/04	771.30	4/2/04	-112.76	-112.79	0.03

The geoid residuals (ΔN) fall within in the following ranges:

Station Group	Min	Max	Average
15 with leveled NAVD 88:	-0.17ft	+0.03ft	-0.06ft
All 34:	-0.20ft	+0.03ft	-0.08ft

This range, spanning about 0.2 feet in the Madison area, can be thought of as how poorly GEOID03 predicts NAVD 88(1991) elevations in the Madison area. Geoid residuals at the 19 stations with GPS-derived NAVD 88(1991) elevations are similar to those at leveled stations, so all 34 stations are considered together here. Figure 10 is a map of the geoid residuals. Notice that geoid residuals generally increase from west to east. Assuming there is one 1997 NAD 83 ellipsoid and two different ways to determine the geoid, Figure 11 shows this east-west trend as a tilt between the true NAVD 88(1991) geoid (as defined by published NGS elevations) and the geoid defined by subtracting (negative) GEOID03 geoid separations (N) from published 1997 NAD 83 ellipsoid heights (h). Note that Figure 11 is not to scale. From west to east, the NAD 83 ellipsoid and NAVD 88 geoid get farther apart by about 0.8 feet, while the geoid residual (between geoids) only increases by about 0.2 feet.

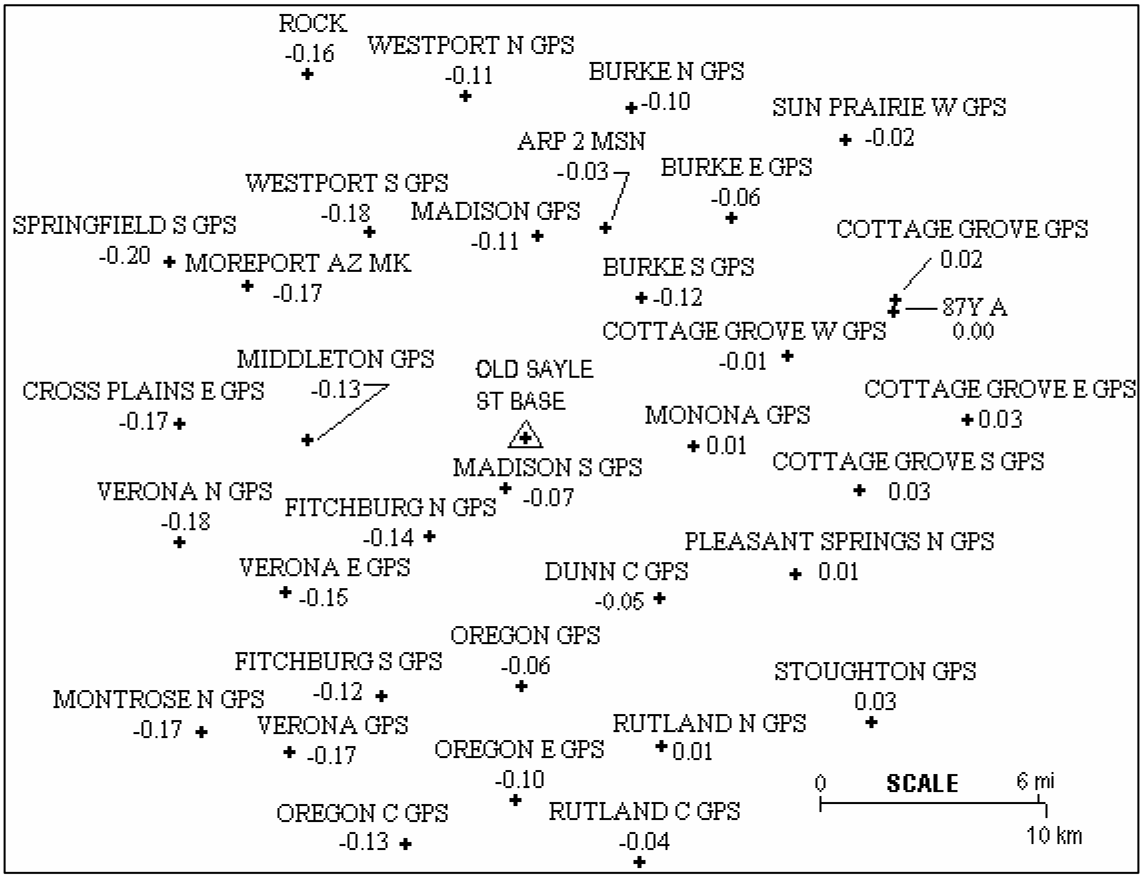


Figure 10: 1997 GEOID03 Residuals, feet
Using 1997 NAD 83 ellipsoid heights and NAVD 88(1991) elevations

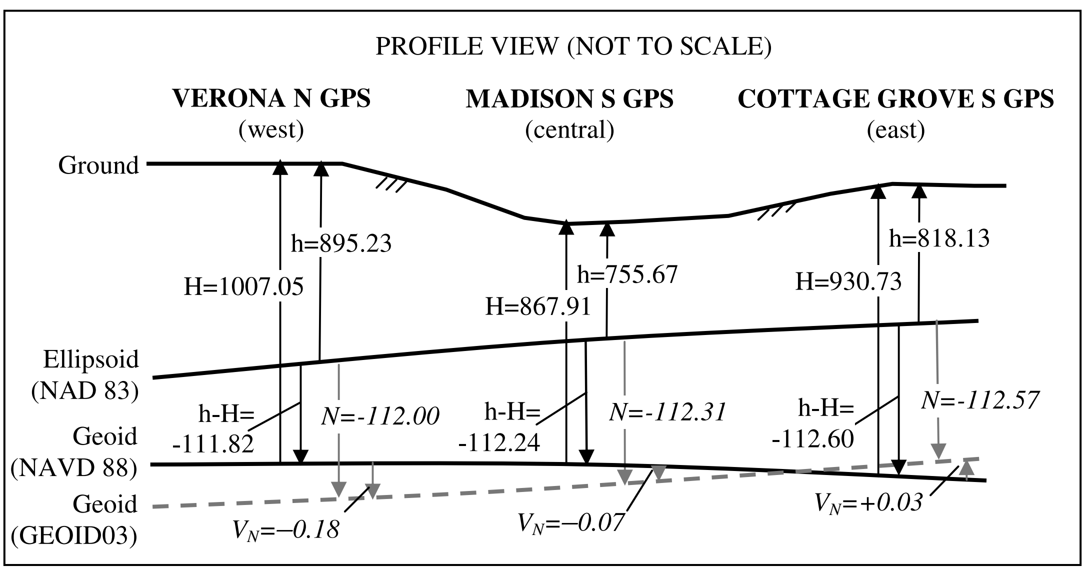


Figure 11: Selected "Old" Geoid Residuals (West, Central and East), feet
H = elevation above NAVD 88(1991) geoid
h = height above 1997 NAD 83 ellipsoid
h-H = geoid separation computed from published H & h
N = geoid separation predicted by GEOID03 geoid model; dashed line datum created by h - N
V_N = geoid residual = N–(h–H)

3.4. SHIFTING AND TILTING GEOID03: Since the geoid residuals follow a more or less systematic spatial pattern, it is possible to modify GEOID03 by vertically shifting and tilting it to better fit the ellipsoid and geoid. The pivot point (origin) used is the old Sayle Street GPS base station, at Latitude 43°03'17.13258", Longitude 89°22'57.67569", Dane County Coordinates Northing 475567.53 US ft, Easting 821567.38 US ft, NAD 83(1997) Datum. Tilts are computed along the Dane County Coordinate System's North and East axes at that point. Note that the convergence angle (difference between geodetic north and grid north) at that point is only +0°01'37", so the tilt values are effectively the same relative to either grid north or geodetic north. The best-fit shift and tilt parameters were solved using unweighted least squares (equations are in Appendix 1 at the end of this document). Figure 12 shows an east-west profile view before the shift and tilt are applied. Figure 13 shows the geoid residuals after shift and tilt.

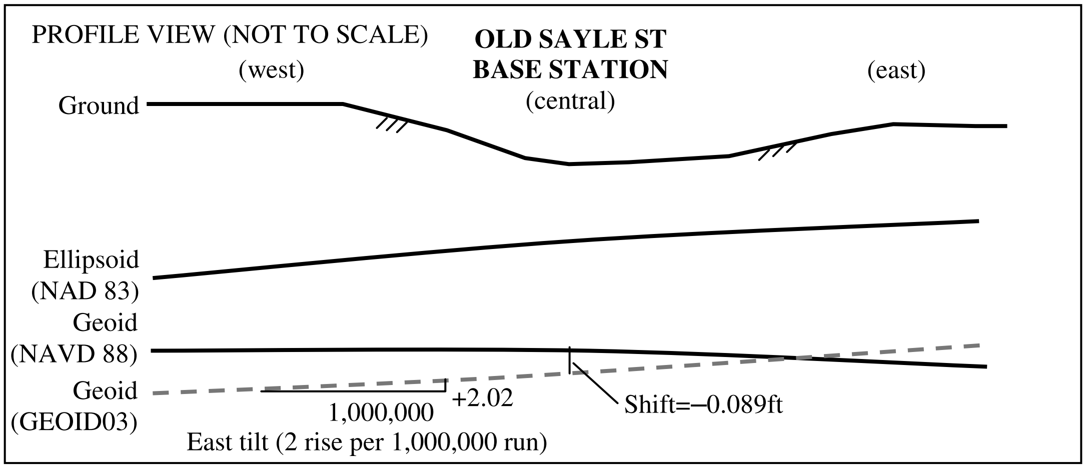


Figure 12: East-West Profile of GEOID03 Before Shift and Tilt Applied, feet

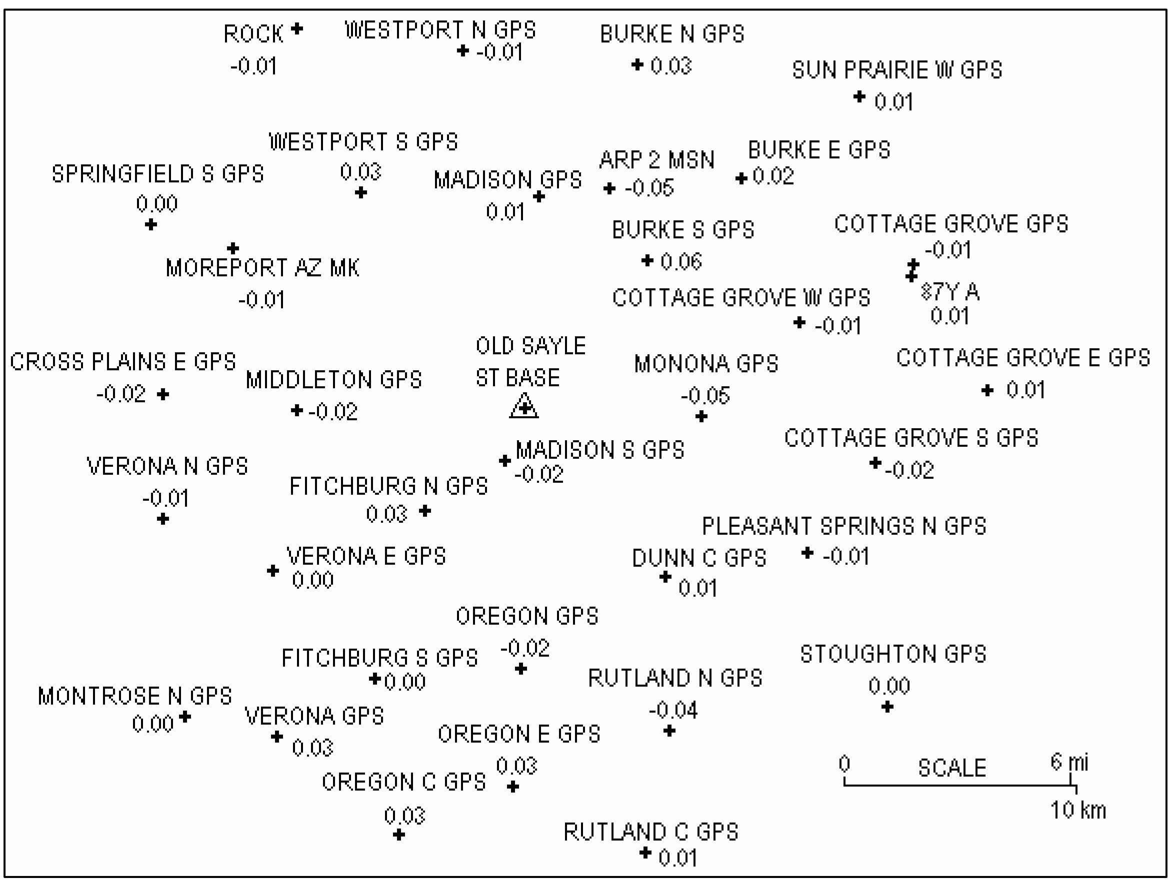


Figure 13: Modified 1997 GEOID03 Residuals After Shift and Tilt, feet

Using 1997 NAD 83 ellipsoid heights and NAVD 88(1991) elevations. Best-fit modifications to GEOID03 (+/- standard deviation) are: Shift at pivot point (old Sayle Street base station) = -0.089 ft (+/- 0.004 ft); North tilt = -0.35 ppm (+/- 0.1 ppm); East tilt = +2.02 ppm (+/- 0.1 ppm)

The modified GEOID03 now fits published 1997 NAD 83 ellipsoid heights and NAVD 88(1991) elevations at the 34 stations within 0.06 feet (88% of stations within 0.03 ft). This is a better fit than the unmodified GEOID03, which had geoid residuals up to 0.20 ft. The best-fit shift at the Madison GPS base station is –0.089 feet, which is very close to the observed geoid residual of -0.11 ft based on independent GPS and leveling observations (Table 20). Note that the north-south tilt (-0.35 parts per million, or ppm) is 1/5th the magnitude of the east-west tilt (+2.02 ppm). A tilt of 0.35 ppm is only 0.02 feet in 12 miles, which is undetectable with all but the most precise GPS techniques. In comparison, a tilt of 2.02 ppm is 0.13 feet in 12 miles, which is probably detectable and worth accounting for. Tilt values are shown to 0.01 ppm, even though standard deviations from the fit are +/-0.1 ppm (10 times larger), only so that users can reproduce the modified geoid residuals with minimal rounding error.

3.5. WHY DOESN'T IT FIT EXACTLY? Even after shifting and tilting GEOID03, it does not perfectly match the 1997 NAD 83 ellipsoid and NAVD 88(1991) geoid surfaces. If it did, the modified geoid residuals (Figure 13) would all be zero. The discrepancy could be due to small errors in the NGS-published ellipsoid heights and NAVD 88 elevations, or simply because a station moved between the dates of the ellipsoid height and NAVD 88 elevation measurements (see ARP 2 MSN in Figure 08). It could also be error in the GEOID03 geoid model. If shifting and tilting doesn't make it fit, it may be the wrong shape. In other words, there may be local curvatures in the true geoid surface (caused by local variations in gravity) that are captured by the leveled NAVD 88 elevations but not by the GEOID03 geoid model. Regardless, the modified geoid residuals are 0.06 feet or less, which is very difficult to detect with any but the most precise GPS procedures.

3.6. 2007 GEOID03 RESIDUALS The same comparison can be made among GEOID03, 2007 NAD 83 ellipsoid heights, and NAVD 88(2007) elevations. The same 34 stations used in Table 04 are evaluated. Note that the 19 stations with GPS-derived NAVD 88 values have NAVD 88(2007) values on NGS data sheets downloaded December 2, 2007, and NAVD 88(1991) values on NGS data sheets downloaded July 30, 2007. However, NGS does not currently (Mar 2013) distinguish between different adjustments of NAVD 88, and the December data sheets do not show the previous GPS-derived NAVD 88 elevations in the "superceded values" section. Also, the ellipsoid height 95%-confidence network accuracy estimates from the NGS data sheets are included below. These are only available for 2007 NAD 83 ellipsoid heights, and they are used here to weight the best-fit geoid shift and tilt computation.

TABLE 05: GEOID03 Geoid Residuals for NAVD 88(2007) Elevations and 2007 NAD 83 Ellipsoid Heights
From NGS Data Sheets retrieved December 2, 2007

Station	**H**: NAVD 88(2007), ft	NAVD 88 source	**h**: 2007 NAD 83 Ellip Ht, ft	Ellip Ht 95% accuracy, ft	**N** (GEOID03), ft	**h-H**, ft	**ΔN** = N-(h-H), ft
MADISON S GPS	868.01	Leveled	755.66	0.017	-112.31	-112.35	0.04
BURKE S GPS	851.26	Leveled	738.42	0.019	-112.86	-112.84	-0.02
OREGON GPS	977.91	Leveled	865.98	0.005	-111.86	-111.93	0.07
FITCHBURG S GPS	1059.74	Leveled	947.94	0.015	-111.76	-111.80	0.04
RUTLAND N GPS	920.43	Leveled	808.42	0.013	-111.88	-112.01	0.13
COTTAGE GROVE S GPS	930.84	Leveled	818.14	0.010	-112.57	-112.70	0.13
BURKE N GPS	884.13	Leveled	770.65	0.011	-113.47	-113.48	0.01
WESTPORT N GPS	937.93	Leveled	824.65	0.012	-113.30	-113.28	-0.02
OREGON E GPS	1034.14	Leveled	922.5	0.014	-111.62	-111.64	0.02
87Y A	901.03	Leveled	787.92	0.008	-113.02	-113.11	0.09
VERONA GPS	952.80	Leveled	841.18	0.013	-111.61	-111.62	0.01
COTTAGE GROVE GPS	901.49	Leveled	788.33	0.005	-113.06	-113.16	0.10
ROCK	1059.6	Leveled	946.46	0.005	-113.20	-113.14	-0.06
SUN PRAIRIE W GPS	961.14	Leveled	847.53	0.011	-113.52	-113.61	0.09
STOUGHTON GPS	888.81	Leveled	776.60	0.015	-112.08	-112.21	0.13
FITCHBURG N GPS	936.38	GPS-derived	824.2	0.029	-112.16	-112.18	0.02
MONONA GPS	863.19	GPS-derived	750.53	0.012	-112.56	-112.66	0.10
MADISON GPS	893.37	GPS-derived	780.42	0.015	-112.93	-112.95	0.02
DUNN C GPS	858.30	GPS-derived	746.00	0.024	-112.21	-112.30	0.09
MIDDLETON GPS	1080.02	GPS-derived	967.75	0.023	-112.25	-112.27	0.02
ARP 2 MSN	855.38	GPS-derived	742.32	0.053	-113.03	-113.06	0.03
WESTPORT S GPS	872.44	GPS-derived	759.72	0.015	-112.78	-112.72	-0.06

Station	**H**: NAVD 88(2007), ft	NAVD 88 source	**h**: 2007 NAD 83 Ellip Ht, ft	Ellip Ht 95% accuracy, ft	**N** (GEOID03), ft	**h-H**, ft	**ΔN** = N-(h-H), ft
COTTAGE GROVE W GPS	891.93	GPS-derived	778.99	0.011	-112.83	-112.94	0.11
VERONA E GPS	953.44	GPS-derived	841.47	0.019	-111.96	-111.97	0.01
BURKE E GPS	925.52	GPS-derived	812.28	0.015	-113.19	-113.24	0.05
PLEASANT SPRINGS N GPS	861.35	GPS-derived	748.85	0.010	-112.37	-112.50	0.13
MOREPORT AZ MK	926.02	GPS-derived	813.53	0.019	-112.54	-112.49	-0.05
CROSS PLAINS E GPS	1077.52	GPS-derived	965.34	0.012	-112.21	-112.18	-0.03
VERONA N GPS	1007.18	GPS-derived	895.21	0.013	-112.00	-111.97	-0.03
SPRINGFIELD S GPS	1058.23	GPS-derived	945.76	0.011	-112.56	-112.47	-0.09
OREGON C GPS	918.21	GPS-derived	806.70	0.014	-111.48	-111.51	0.03
MONTROSE N GPS	927.20	GPS-derived	815.62	0.012	-111.60	-111.58	-0.02
RUTLAND C GPS	914.99	GPS-derived	803.27	0.019	-111.63	-111.72	0.09
COTTAGE GROVE E GPS	884.18	GPS-derived	771.32	0.010	-112.76	-112.86	0.10

The NAVD 88(2007) – 2007 NAD 83 geoid residuals for these same 34 stations lie in the following range:

Min	Max	Average
-0.09ft	+0.13ft	+0.04ft

Note that the average 2007 geoid residual (+0.04ft) is 0.12ft larger than the average 1997 geoid residual (-0.08 ft) (Table 04). This makes sense, because while ellipsoid heights of Madison area stations did not change much from 1997 to 2007, NAVD 88 elevations did increase an average of 0.11 ft from 1991to 2007 (Figure 02 and Figure 03). The geoid residual formula is ΔN=N–(h–H), so if the NAVD 88 elevation (H) increases, the geoid residual (ΔN) increases.

Figure 14 shows that the 2007 geoid residuals follow a similar trend as the 1997 geoid residuals, generally increasing from west to east. Figure 15 shows the modified geoid residuals after GEOID03 has been shifted and tilted to best-fit the 2007 NAD 83 ellipsoid and NAVD 88(2007) geoid. This best-fit computation was weighted by the reciprocal of the square of the ellipsoid height 95%-confidence network accuracy estimates from the NGS data sheets. Note higher modified residuals at BURKE S GPS (0.06 ft) and OREGON E GPS (0.05 ft).

Table 06 compares the 2007 and 1997 shift and tilt parameters (Figure 13 and Figure 15). Note that the difference in the shifts (+0.033 – (-0.089)) = 0.12 ft, closely matches the average 0.11 ft shift from 1991 to 2007 NAVD 88 elevations (Figure 03). The change in tilt values reflects the slight tilts seen in both the readjustment of NAVD 88 elevations (Figure 03) and NAD 83 ellipsoid heights (Figure 08). Note that both of the tilt values only changed by about 0.5 ppm, which is only detectable by the most precise GPS techniques.

Table 06: Comparison of 2007 and 1997 GEOID03 Shift and Tilt Parameters
Standard deviations from least-squares adjustment shown in parentheses.

Parameter	2007 (Figure 15)	1997 (Figure 13)
Shift at old Sayle Street Base	+0.033 ft (+/- 0.003 ft)	-0.089 ft (+/- 0.004 ft)
East-West Tilt	+1.58 ppm (+/- 0.1 ppm)	+2.02 ppm (+/- 0.1 ppm)
North-South Tilt	-0.83 ppm (+/- 0.1 ppm)	-0.35 ppm (+/- 0.1 ppm)

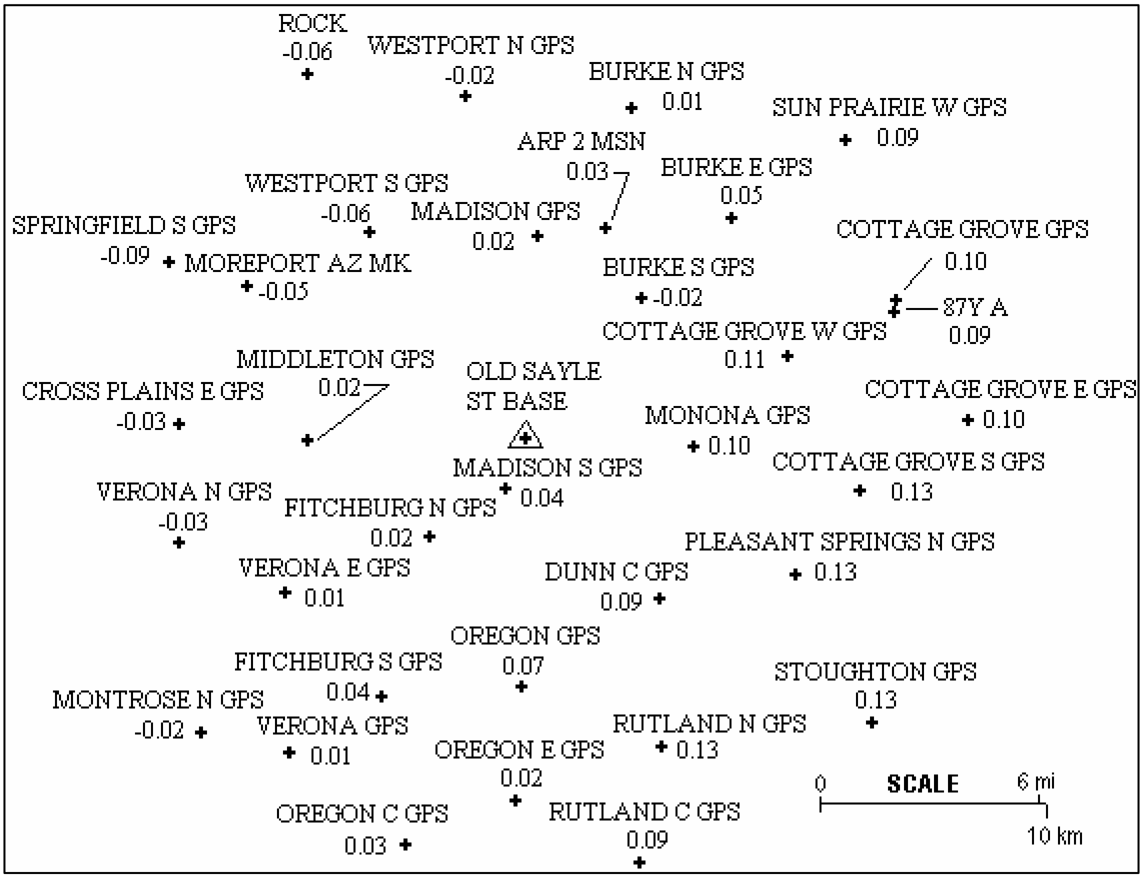


Figure 14: 2007 GEOID03 Residuals, feet Using 2007 NAD 83 ellipsoid heights and NAVD 88(2007) elevations

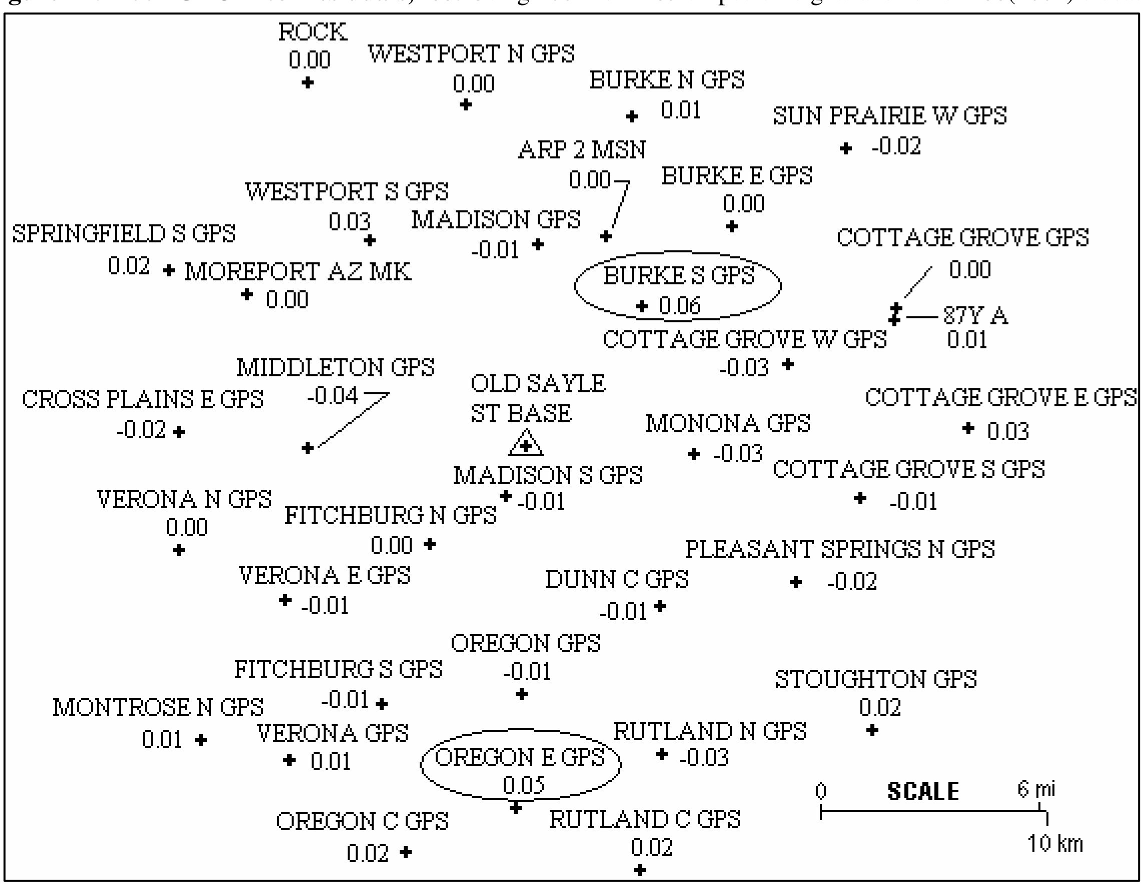


Figure 15: Modified 2007 GEOID03 Residuals After Shift and Tilt, feet Using 2007 NAD 83 ellipsoid heights and NAVD 88(2007) elevations. Best-fit modifications to GEOID03 (+/- standard deviation): Shift at pivot point (old Sayle Street base station) = +0.033 ft (+/- 0.003 ft); North tilt = -0.83 ppm (+/- 0.1 ppm); East tilt = +1.58 ppm (+/- 0.1 ppm)

3.7. 2007 / 1991 GEOID03 RESIDUALS Similarly, The GEOID03 geoid model can be compared to 2007 NAD 83 ellipsoid heights and NAVD 88(1991) elevations, which is useful for preserving the NAVD 88(1991) datum used for elevations published at City of Madison Public Land Survey section corners. Table 07 shows the data for the same 34 stations used for the 1997 and 2007 comparison (Table 04):

TABLE 07: GEOID03 Geoid Residuals for NAVD 88(1991) and 2007 NAD 83 Ellipsoid Heights
From NGS Data Sheets retrieved July 31, 2007, and December 2, 2007

Station	H NAVD 88(1991), ft	NAVD 88 source	NAVD 88 Date	h 2007 NAD 83 Ellip Ht, ft	N (GEOID03), ft	h-H, ft	ΔN = N-(h-H), ft
MADISON S GPS	867.91	Leveling	02/25/04	755.66	-112.31	-112.25	-0.06
BURKE S GPS	851.16	Leveling	02/25/04	738.42	-112.86	-112.74	-0.12
OREGON GPS	977.81	Leveling	02/25/04	865.98	-111.86	-111.83	-0.03
FITCHBURG S GPS	1059.61	Leveling	02/25/04	947.94	-111.76	-111.67	-0.09
RUTLAND N GPS	920.33	Leveling	02/25/04	808.42	-111.88	-111.91	0.03
COTTAGE GROVE S GPS	930.73	Leveling	02/25/04	818.14	-112.57	-112.59	0.02
BURKE N GPS	884.02	Leveling	02/25/04	770.65	-113.47	-113.37	-0.10
WESTPORT N GPS	937.82	Leveling	02/25/04	824.65	-113.30	-113.17	-0.13
OREGON E GPS	1034.06	Leveling	02/25/04	922.50	-111.62	-111.56	-0.06
87Y A	900.92	Leveling	02/25/04	787.92	-113.02	-113.00	-0.02
VERONA GPS	952.66	Leveling	02/25/04	841.18	-111.61	-111.48	-0.13
COTTAGE GROVE GPS	901.38	Leveling	02/25/04	788.33	-113.06	-113.05	-0.01
ROCK	1059.48	Leveling	02/25/04	946.46	-113.20	-113.02	-0.18
SUN PRAIRIE W GPS	961.02	Leveling	02/25/04	847.53	-113.52	-113.49	-0.03
STOUGHTON GPS	888.72	Leveling	02/25/04	776.60	-112.08	-112.12	0.04
FITCHBURG N GPS	936.25	GPS-derived	4/2/04	824.20	-112.16	-112.05	-0.11
MONONA GPS	863.09	GPS-derived	4/2/04	750.53	-112.56	-112.56	0.00
MADISON GPS	893.24	GPS-derived	4/2/04	780.42	-112.93	-112.82	-0.11
DUNN C GPS	858.17	GPS-derived	4/2/04	746.00	-112.21	-112.17	-0.04
MIDDLETON GPS	1079.89	GPS-derived	4/2/04	967.75	-112.25	-112.14	-0.11
ARP 2 MSN	855.38	GPS-derived	4/28/99	742.32	-113.03	-113.06	0.03
WESTPORT S GPS	872.34	GPS-derived	4/2/04	759.72	-112.78	-112.62	-0.16
COTTAGE GROVE W GPS	891.80	GPS-derived	4/2/04	778.99	-112.83	-112.81	-0.02
VERONA E GPS	953.31	GPS-derived	4/2/04	841.47	-111.96	-111.84	-0.12
BURKE E GPS	925.39	GPS-derived	4/2/04	812.28	-113.19	-113.11	-0.08
PLEASANT SPRINGS N GPS	861.22	GPS-derived	4/2/04	748.85	-112.37	-112.37	0.00
MOREPORT AZ MK	925.88	GPS-derived	9/30/02	813.53	-112.54	-112.35	-0.19
CROSS PLAINS E GPS	1077.39	GPS-derived	4/2/04	965.34	-112.21	-112.05	-0.16
VERONA N GPS	1007.05	GPS-derived	4/2/04	895.21	-112.00	-111.84	-0.16
SPRINGFIELD S GPS	1058.13	GPS-derived	4/2/04	945.76	-112.56	-112.37	-0.19
OREGON C GPS	918.08	GPS-derived	4/2/04	806.70	-111.48	-111.38	-0.10
MONTROSE N GPS	927.07	GPS-derived	4/2/04	815.62	-111.60	-111.45	-0.15
RUTLAND C GPS	914.89	GPS-derived	4/2/04	803.27	-111.63	-111.62	-0.01
COTTAGE GROVE E GPS	884.09	GPS-derived	4/2/04	771.32	-112.76	-112.77	0.01

The range of geoid residuals is similar to the NAVD 88(1991) – 1997 NAD 83 ellipsoid data set (Table 04).

Data Set		Min	Max	Average
NAVD 88(1991) & 2007 NAD 83	-0.19ft	+0.04ft	-0.08ft	(Table 07)
NAVD 88(1991) & 1997 NAD 83	*-0.20ft*	*+0.03ft*	*-0.08ft*	*(Table 04)*

Figure 16 shows the geoid residuals, with the same east-west trend. Note that the geoid residual at station ARP 2 MSN (PID #OM1387) is significantly different than its neighbors. Its NAVD 88(1991) elevation was derived from

GPS observations on 04/28/99, while the 2007 NAD 83 ellipsoid height is probably from more recent measurements. Table 03 shows that ARP 2 MSN's 2007 NAD 83 ellipsoid height is 0.06 feet less than its 1997 NAD 83 ellipsoid height, and this station is listed on the NGS data sheet as "stability type C", which is "of type commonly subject to surface motion." Station ARP 2 MSN appears to have sunk a little.

Figure 17 shows the modified geoid residuals after a best-fit shift and tilt of GEOID03. Ellipsoid height 95%-confidence network accuracy estimates (Table 05) were again used to weight the solution. Note that ARP 2 MSN's network accuracy estimate is 0.05 ft, significantly larger than for other stations. After the best-fit, ARP2 MSN's modified residual was –0.12 ft, confirming its poor fit with the rest of the stations. A test of much lower weights for ARP 2 MSN did not change any residuals, confirming that it is not adversely affecting the overall solution. The next highest modified residual is at BURKE S GPS (0.05 ft), which also had a higher modified residual using the 2007 values (Figure 15).

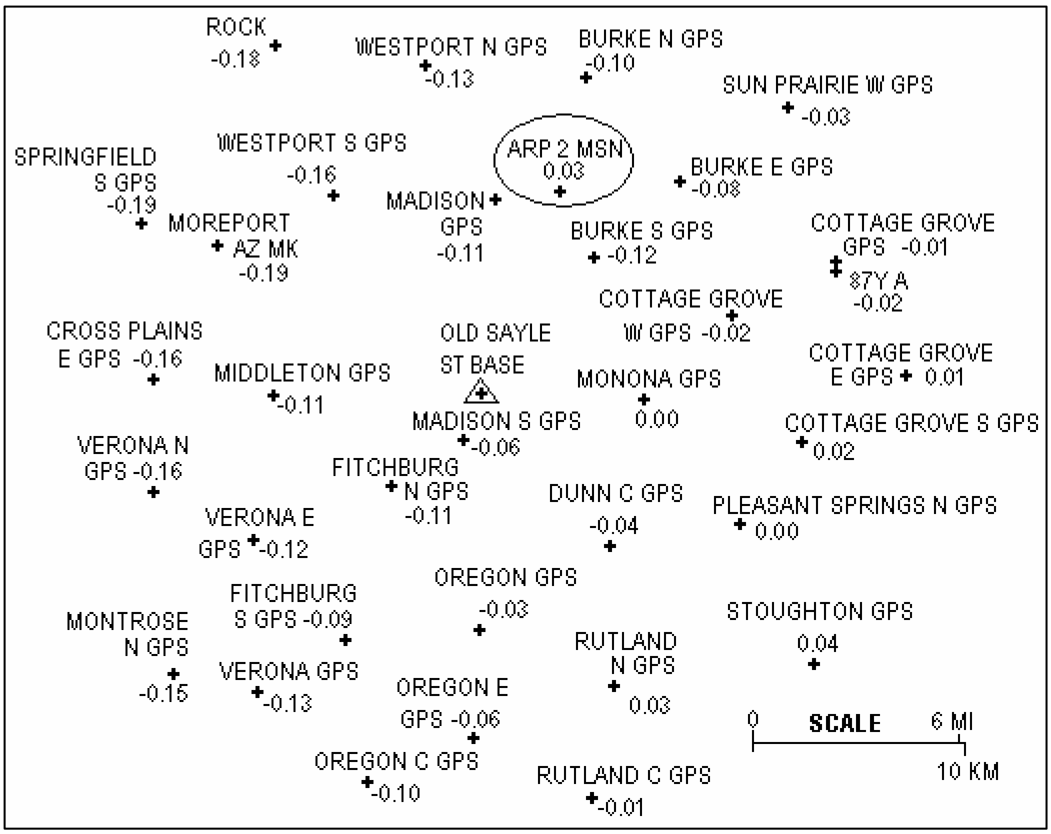


Figure 16: 1991 / 2007 GEOID03 Residuals, feet
Using 2007 NAD 83 ellipsoid heights and NAVD 88(1991) elevations

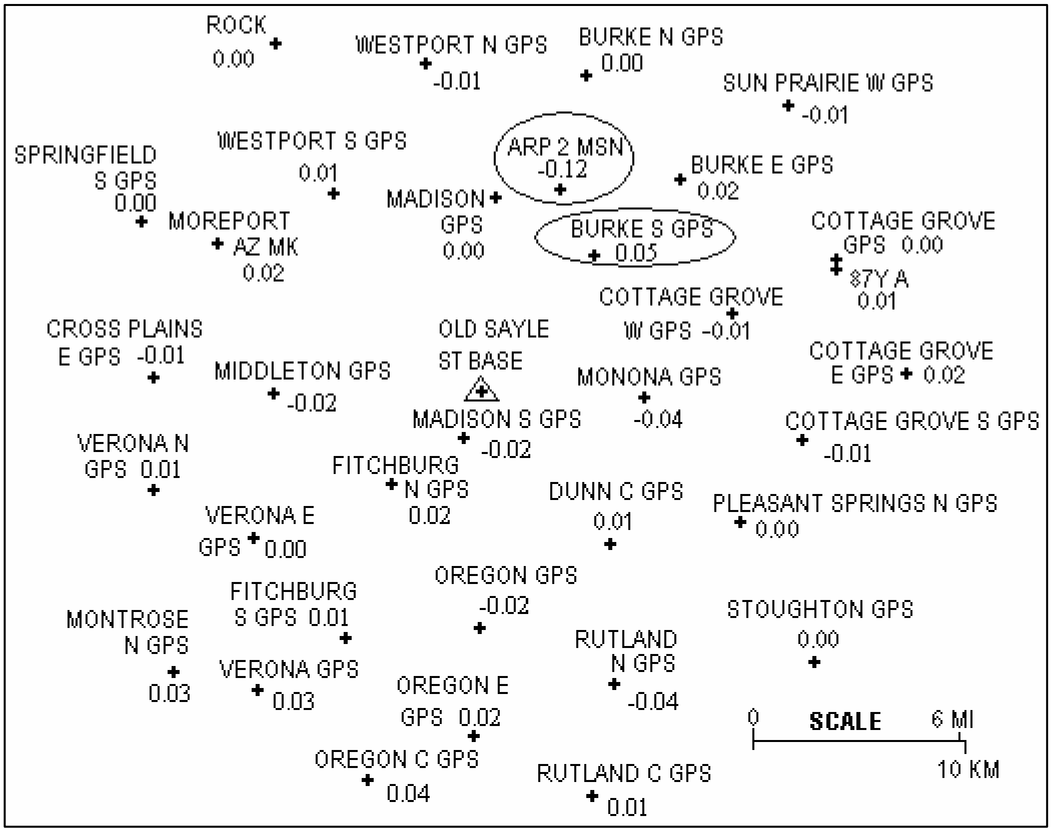


Figure 17: 1991/2007 Modified GEOID03 Residuals After Shift and Tilt, feet Using 2007 NAD 83 ellipsoid heights and NAVD 88(1991) elevations. Best-fit modifications to GEOID03 (+/- standard deviation): Shift at pivot point (old Sayle St base) = -0.081 ft (+/- 0.003 ft); North tilt = -0.90 ppm (+/- 0.1 ppm); East tilt = +1.70 ppm (+/- 0.1 ppm)

3.8. COMPARISON & CONVERSIONS (ORIGIN, STATE PLANE)

Table 08 compares the shifts and tilts for the three solutions computed so far. Comparing 2007 NAD 83 - NAVD 88(1991) (first column) and 1997 NAD 83 - NAVD 88(1991) (third column), the parameters are almost identical, except that the north-south tilt is slightly more significant. This is because of the generally southwest-northeast tilt in the ellipsoid height adjustment from 1997 to 2007 (Figure 08).

Table 08: Comparison of GEOID03 Shift and Tilt Parameters
Standard deviations from least-squares adjustment shown in parentheses.

Parameter	2007 NAD 83, NAVD 88(1991) (Figure 17)	2007 NAD 83, NAVD 88(2007) (Figure 15)	1997 NAD 83, NAVD 88(1991) (Figure 13)
Shift at Sayle Street Base	-0.081 ft (+/- 0.003 ft)	+0.033 ft (+/- 0.003 ft)	-0.089 ft (+/- 0.004 ft)
East-West Tilt	+1.70 ppm (+/- 0.1 ppm)	+1.58 ppm (+/- 0.1 ppm)	+2.02 ppm (+/- 0.1 ppm)
North-South Tilt	**-0.90 ppm** (+/- 0.1 ppm)	-0.83 ppm (+/- 0.1 ppm)	**-0.35 ppm** (+/- 0.1 ppm)

The “shift” values in Table 08 are computed for an origin / pivot point at the old Sayle Street base station. With the base station move to Emil Street in 2009, the above parameters need not be changed, since they are independent of the actual GPS base station location. However, it is useful to see the shift value at the current base station to easily compare its NAD 83 ellipsoid height and NAVD 88 elevation.

It should be noted that the tilt values are independent of the origin location. Also, since the tilt values are very small (maximum about 2 ppm, or about 0.01 ft per mile), only approximate horizontal coordinates (+/- 50 ft) are needed for the origin.

Below are the computations for moving the origin from the Sayle Street base station to the Emil Street base station, using the parameters from the 2007 NAD 83 ellipsoid and NAVD 88(2007) geoid. Horizontal coordinates are Dane County Coordinates, NAD 83 Datum (datum adjustment irrelevant since only an approximate horizontal position is required):

Sayle Street base: N = 475568 US ft E = 821567 US ft
Emil Street base: N = 469050 US ft E = 814396 US ft
Sayle to Emil: dN = -6518 ft dE = -7171 ft
2007-2007 tilt values: tiltN = -0.83 ppm tiltE = +1.58 ppm
North tilt component of geoid residual (Sayle to Emil) = (dN x tiltN) = (-6518 x -0.83/1000000) = +0.005 ft
East tilt component of geoid residual (Sayle to Emil) = (dE x tiltE) = (-7171 x +1.58/1000000) = -0.011 ft
Shift component of geoid residual at Sayle St = +0.033 ft (Table 08)

Geoid residual at Emil St = sum of 3 components = (0.005 -0.011 +0.033) = +0.027 ft
Shift parameter = geoid residual = +0.027 ft

Table 09: GEOID03 Shift and Tilt Parameters: Origin Translated to Emil Street Base Station
Approximate Dane County Coordinates, NAD 83 Datum, of Emil Street base are N=469050 US ft, E=814396 US ft
Values in parentheses are standard deviations from least-squares best-fit computation of shift & tilt parameters.

Parameter	2007 NAD 83, NAVD 88(1991) (Figure 17)	2007 NAD 83, NAVD 88(2007) (Figure 15)	1997 NAD 83, NAVD 88(1991) (Figure 13)
Shift at Emil Street Base	**-0.087 ft** (+/- 0.003 ft)	**+0.027 ft** (+/- 0.003 ft)	**-0.101 ft** (+/- 0.004 ft)
East-West Tilt	+1.70 ppm (+/- 0.1 ppm)	+1.58 ppm (+/- 0.1 ppm)	+2.02 ppm (+/- 0.1 ppm)
North-South Tilt	-0.90 ppm (+/- 0.1 ppm)	-0.83 ppm (+/- 0.1 ppm)	-0.35 ppm (+/- 0.1 ppm)

STATE PLANE: The geoid shift and tilt parameters above were computed using horizontal positions in Dane County Coordinates, NAD 83 Datum. Note that datum adjustment and difference between WISCRS and WCCS county coordinate projections are insignificant for these calculations. Table 10 shows the geoid shift and tilt parameters computed using horizontal positions in the Wisconsin Plane Coordinate System, South Zone, NAD 83 Datum. The State Plane system is slightly rotated (approx. 0°23’50”) and scaled (approx. 1 part in 10,000) relative to the Dane County system in the Madison area. The resulting difference is seen in the tilt parameters (Table 10), and only on the order of 0.01 ppm, or 0.01 foot in 200 miles, which is insignificant relative to the residuals in the best-fit geoid shift and tilt computation (Figure 13, Figure 15 & Figure 17). The shift and tilt values in Table 09 can be used with either Dane County Coordinates or State Plane Coordinates, NAD 83 Datum, if the origin / pivot point is given coordinates in the same system.

Table 10: GEOID03 Shift and Tilt Parameters: Wisconsin State Plane Coordinates, NAD 83 Datum
Approximate WI State Plane (South Zone) Coordinates, NAD 83 Datum, of the Emil Street base station are N=378471 US ft, E=2126363 US ft. Values in parentheses are changes from tilts relative to Dane County Coordinate axes (Table 09).

Parameter	2007 NAD 83, NAVD 88(1991) (Figure 17)	2007 NAD 83, NAVD 88(2007) (Figure 15)	1997 NAD 83, NAVD 88(1991) (Figure 13)
Shift at Emil Street Base	-0.087 ft	+0.027 ft	-0.101 ft
East-West Tilt	+1.71 ppm **(+0.01 ppm)**	+1.58 ppm	+2.02 ppm
North-South Tilt	-0.89 ppm **(+0.01 ppm)**	-0.82 ppm **(+0.01 ppm)**	-0.34 ppm **(+0..01 ppm)**

3.9. GEOID09:

In 2009 the NGS released the GEOID09 geoid model. Table 11 shows the same geoid residual tables as produced above (Table 05, Figure 14 & Figure 15), except using GEOID09 instead of GEOID03.

Table 11: GEOID09 Geoid Residuals for NAVD 88(2007) Elevations and 2007 NAD 83 Ellipsoid Heights
From NGS Data Sheets retrieved December 2, 2007

Station	**H**: NAVD 88(2007), ft	NAVD 88 source	**h**: 2007 NAD 83 Ellip Ht, ft	Ellip Ht 95% accuracy, ft	**N** (GEOID09), ft	**h-H**, ft	**ΔN =** N-(h-H), ft
MADISON S GPS	*868.01*	*Leveled*	*755.66*	*0.017*	-112.32	*-112.35*	0.03
BURKE S GPS	*851.26*	*Leveled*	*738.42*	*0.019*	-112.86	*-112.84*	-0.02
OREGON GPS	*977.91*	*Leveled*	*865.98*	*0.005*	-111.92	*-111.93*	0.01
FITCHBURG S GPS	*1059.74*	*Leveled*	*947.94*	*0.015*	-111.79	*-111.80*	0.01
RUTLAND N GPS	*920.43*	*Leveled*	*808.42*	*0.013*	-112.00	*-112.01*	0.01
COTTAGE GROVE S GPS	*930.84*	*Leveled*	*818.14*	*0.010*	-112.69	*-112.70*	0.01
BURKE N GPS	*884.13*	*Leveled*	*770.65*	*0.011*	-113.49	*-113.48*	-0.01
WESTPORT N GPS	*937.93*	*Leveled*	*824.65*	*0.012*	-113.30	*-113.28*	-0.02
OREGON E GPS	*1034.14*	*Leveled*	*922.50*	*0.014*	-111.66	*-111.64*	-0.02
87Y A	*901.03*	*Leveled*	*787.92*	*0.008*	-113.11	*-113.11*	0.00
VERONA GPS	*952.80*	*Leveled*	*841.18*	*0.013*	-111.61	*-111.62*	0.01
COTTAGE GROVE GPS	*901.49*	*Leveled*	*788.33*	*0.005*	-113.15	*-113.16*	0.01
ROCK	*1059.6*	*Leveled*	*946.46*	*0.005*	-113.12	*-113.14*	0.02
SUN PRAIRIE W GPS	*961.14*	*Leveled*	*847.53*	*0.011*	-113.59	*-113.61*	0.02
STOUGHTON GPS	*888.81*	*Leveled*	*776.60*	*0.015*	-112.23	*-112.21*	-0.02
FITCHBURG N GPS	*936.38*	*GPS-derived*	*824.20*	*0.029*	-112.18	*-112.18*	0.00
MONONA GPS	*863.19*	*GPS-derived*	*750.53*	*0.012*	-112.61	*-112.66*	0.05
MADISON GPS	*893.37*	*GPS-derived*	*780.42*	*0.015*	-112.91	*-112.95*	0.04
DUNN C GPS	*858.30*	*GPS-derived*	*746.00*	*0.024*	-112.29	*-112.30*	0.01
MIDDLETON GPS	*1080.02*	*GPS-derived*	*967.75*	*0.023*	-112.22	*-112.27*	0.05
ARP 2 MSN	*855.38*	*GPS-derived*	*742.32*	*0.053*	-113.02	*-113.06*	0.04
WESTPORT S GPS	*872.44*	*GPS-derived*	*759.72*	*0.015*	-112.70	*-112.72*	0.02
COTTAGE GROVE W GPS	*891.93*	*GPS-derived*	*778.99*	*0.011*	-112.90	*-112.94*	0.04
VERONA E GPS	*953.44*	*GPS-derived*	*841.47*	*0.019*	-111.97	*-111.97*	0.00
BURKE E GPS	*925.52*	*GPS-derived*	*812.28*	*0.015*	-113.22	*-113.24*	0.02
PLEASANT SPRINGS N GPS	*861.35*	*GPS-derived*	*748.85*	*0.010*	-112.49	*-112.50*	0.01
MOREPORT AZ MK	*926.02*	*GPS-derived*	*813.53*	*0.019*	-112.43	*-112.49*	0.06
CROSS PLAINS E GPS	*1077.52*	*GPS-derived*	*965.34*	*0.012*	-112.15	*-112.18*	0.03
VERONA N GPS	*1007.18*	*GPS-derived*	*895.21*	*0.013*	-111.98	*-111.97*	-0.01
SPRINGFIELD S GPS	*1058.23*	*GPS-derived*	*945.76*	*0.011*	-112.46	*-112.47*	0.01
OREGON C GPS	*918.21*	*GPS-derived*	*806.70*	*0.014*	-111.49	*-111.51*	0.02
MONTROSE N GPS	*927.20*	*GPS-derived*	*815.62*	*0.012*	-111.59	*-111.58*	-0.01
RUTLAND C GPS	*914.99*	*GPS-derived*	*803.27*	*0.019*	-111.75	*-111.72*	-0.03
COTTAGE GROVE E GPS	*884.18*	*GPS-derived*	*771.32*	*0.010*	-112.86	*-112.86*	0.00

The NAVD 88(2007) – 2007 NAD 83 geoid residuals using GEOID09 for these 34 stations lie between -0.03ft and +0.06ft, averaging +0.01ft. The geoid residuals (ΔN) in **Table 11** above are close to zero, confirming that the NGS used NAVD 88(2007) elevations to help calibrate GEOID09.

Similarly, Table 12 reproduces the NAVD 88(1991) – 2007 NAD 83 geoid residuals (see Table 07, Figure 16 & Figure 17), except using GEOID09 instead of GEOID03.

Table 12: GEOID09 Geoid Residuals for NAVD 88(1991) Elevations and 2007 NAD 83 Ellipsoid Heights
From NGS Data Sheets retrieved July 31, 2007, and December 2, 2007

Station	H NAVD 88(1991), ft	NAVD 88 source	NAVD 88 Date	h (2007 NAD 83 Ellip Ht), ft	N (GEOID09), ft	h-H, ft	ΔN = N-(h-H), ft
MADISON S GPS	867.91	Leveling	02/25/04	755.66	-112.32	-112.25	-0.07
BURKE S GPS	851.16	Leveling	02/25/04	738.42	-112.86	-112.74	-0.12
OREGON GPS	977.81	Leveling	02/25/04	865.98	-111.92	-111.83	-0.09
FITCHBURG S GPS	1059.61	Leveling	02/25/04	947.94	-111.79	-111.67	-0.12
RUTLAND N GPS	920.33	Leveling	02/25/04	808.42	-112.00	-111.91	-0.09
COTTAGE GROVE S GPS	930.73	Leveling	02/25/04	818.14	-112.69	-112.59	-0.10
BURKE N GPS	884.02	Leveling	02/25/04	770.65	-113.49	-113.37	-0.12
WESTPORT N GPS	937.82	Leveling	02/25/04	824.65	-113.30	-113.17	-0.13
OREGON E GPS	1034.06	Leveling	02/25/04	922.50	-111.66	-111.56	-0.10
87Y A	900.92	Leveling	02/25/04	787.92	-113.11	-113	-0.11
VERONA GPS	952.66	Leveling	02/25/04	841.18	-111.61	-111.48	-0.13
COTTAGE GROVE GPS	901.38	Leveling	02/25/04	788.33	-113.15	-113.05	-0.10
ROCK	1059.48	Leveling	02/25/04	946.46	-113.12	-113.02	-0.10
SUN PRAIRIE W GPS	961.02	Leveling	02/25/04	847.53	-113.59	-113.49	-0.10
STOUGHTON GPS	888.72	Leveling	02/25/04	776.60	-112.23	-112.12	-0.11
FITCHBURG N GPS	936.25	GPS-derived	4/2/04	824.20	-112.18	-112.05	-0.13
MONONA GPS	863.09	GPS-derived	4/2/04	750.53	-112.61	-112.56	-0.05
MADISON GPS	893.24	GPS-derived	4/2/04	780.42	-112.91	-112.82	-0.09
DUNN C GPS	858.17	GPS-derived	4/2/04	746.00	-112.29	-112.17	-0.12
MIDDLETON GPS	1079.89	GPS-derived	4/2/04	967.75	-112.22	-112.14	-0.08
ARP 2 MSN	***855.38***	GPS-derived	4/28/99	742.32	-113.02	-113.06	***0.04***
WESTPORT S GPS	872.34	GPS-derived	4/2/04	759.72	-112.70	-112.62	-0.08
COTTAGE GROVE W GPS	891.80	GPS-derived	4/2/04	778.99	-112.90	-112.81	-0.09
VERONA E GPS	953.31	GPS-derived	4/2/04	841.47	-111.97	-111.84	-0.13
BURKE E GPS	925.39	GPS-derived	4/2/04	812.28	-113.22	-113.11	-0.11
PLEASANT SPRINGS N GPS	861.22	GPS-derived	4/2/04	748.85	-112.49	-112.37	-0.12
MOREPORT AZ MK	925.88	GPS-derived	9/30/02	813.53	-112.43	-112.35	-0.08
CROSS PLAINS E GPS	1077.39	GPS-derived	4/2/04	965.34	-112.15	-112.05	-0.10
VERONA N GPS	1007.05	GPS-derived	4/2/04	895.21	-111.98	-111.84	-0.14
SPRINGFIELD S GPS	1058.13	GPS-derived	4/2/04	945.76	-112.46	-112.37	-0.09
OREGON C GPS	918.08	GPS-derived	4/2/04	806.70	-111.49	-111.38	-0.11
MONTROSE N GPS	927.07	GPS-derived	4/2/04	815.62	-111.59	-111.45	-0.14
RUTLAND C GPS	914.89	GPS-derived	4/2/04	803.27	-111.75	-111.62	-0.13
COTTAGE GROVE E GPS	884.09	GPS-derived	4/2/04	771.32	-112.86	-112.77	-0.09

The NAVD 88(1991) – 2007 NAD 83 geoid residuals (ΔN) using GEOID09 for these 34 stations lie between -0.14ft and -0.05ft (*ARP 2 MSN +0.04ft*), averaging -0.11 ft excluding ARP 2 MSN *(-0.10 ft including ARP 2 MSN)*

Again, Station ARP 2 MSN (PID #OM1387) is an outlier, as in Table 07, indicating that the station appears to have sunk after 1999. The average geoid residual is –0.11 ft, which is the average shift between NAVD 88(1991) and NAVD 88(2007) in the Madison area (Figure 02).

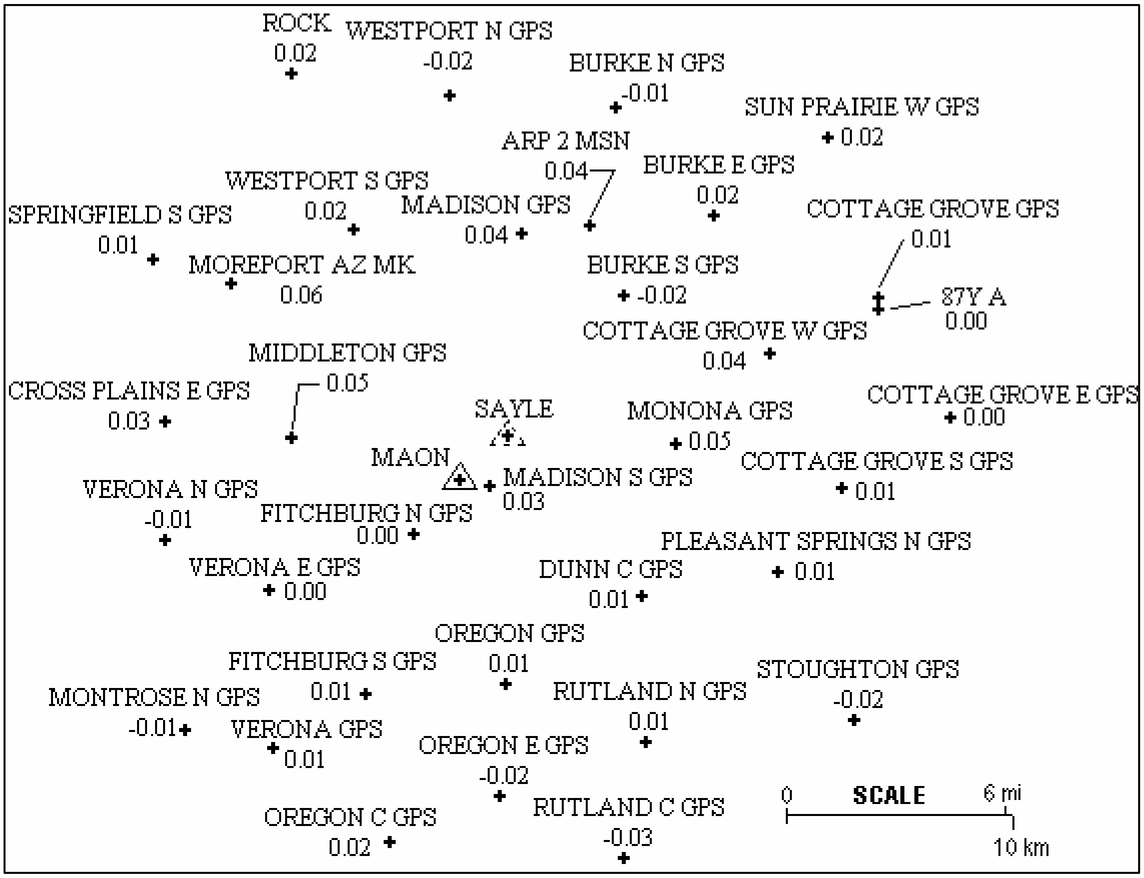


Figure 18: 2007 GEOID09 Residuals, feet Using 2007 NAD 83 ellipsoid heights and NAVD 88(2007) elevations

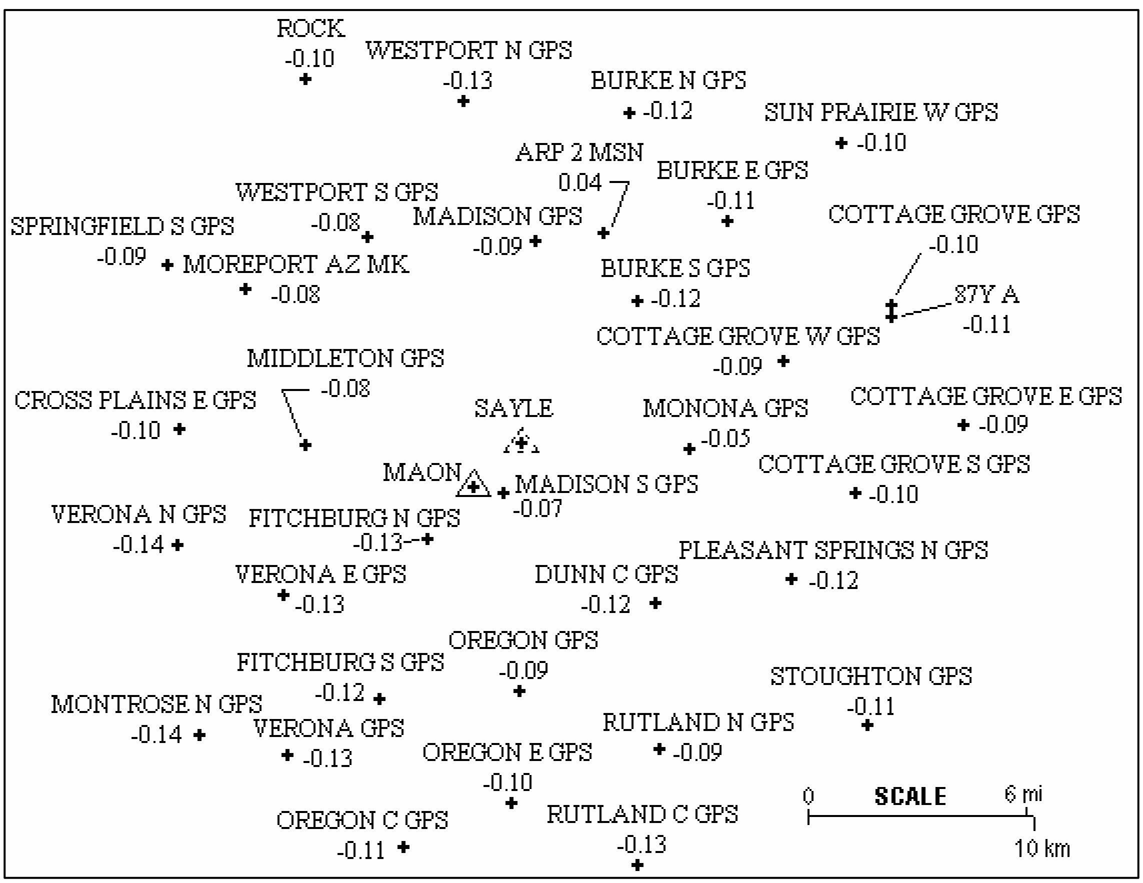


Figure 19: 1991 / 2007 GEOID09 Residuals, feet

Using 2007 NAD 83 ellipsoid heights and NAVD 88(1991) elevations

Table 13 shows the best-fit shift and tilt parameters to minimize the GEOID09 geoid residuals:

Table 13: GEOID09 Shift and Tilt Parameters
Approximate Dane County Coordinates, NAD 83 Datum, of Emil Street base are N=469050 US ft, E=814396 US ft
Values in parentheses are standard deviations from least-squares best-fit computation of shift & tilt parameters.

Parameter	2007 NAD 83, NAVD 88(1991) (Table 12, Figure 19)	2007 NAD 83, NAVD 88(2007) (Table 11, Figure 18)	1997 NAD 83, NAVD 88(1991)
Shift at Emil Street Base	-0.103ft (+/-0.003ft)	+0.011ft (+/-0.003ft)	*not computed*
East-West Tilt	+0.07ppm (+/-0.08ppm)	-0.06ppm (+/-0.07ppm)	*not computed*
North-South Tilt	+0.04ppm (+/-0.09ppm)	+0.11ppm (+/-0.08ppm)	*not computed*

The tilts are similar in magnitude to their standard deviations, indicating no clear improvement by applying tilt corrections to GEOID09 for either data set. 0.1 ppm is 0.01 foot in 19 miles, which is an insignificant correction for all but the most precise GPS techniques.

The shift value for the NAVD 88(2007) – 2007 NAD 83 data is only 0.01 ft, also a correction generally not worth applying. The shift for the NAVD 88(1991) - 2007 NAD 83 data is –0.10 ft, matching the general shift seen between NAVD 88(1991) and NAVD 88(2007) in the Madison area (Figure 02).

Adjusted geoid residuals (after applying shift and tilts) for the 34 stations still reach magnitudes of 0.04 ft (NAVD 88(2007) - 2007 NAD 83) and 0.05 ft (NAVD 88(1991) - 2007 NAD 83, excluding ARP 2 MSN), a reminder that some amount of random error in NGS-published ellipsoid heights, NAVD 88 elevations and/or geoid models is present. The outlier station ARP 2 MSN ends up with a much larger adjusted geoid residual (-0.14 ft) in the NAVD 88(1991) - 2007 NAD 83 data, but it does not significantly affect the shift and tilt computations because each station is weighted by the NGS's published ellipsoid height 95% accuracy. ARP 2 MSN's published accuracy is about 5 times worse than average.

GEOID09 Results: Thus, in the Madison area, to compute elevations consistent with RTK GPS accuracy from the Madison base station, GEOID09 can be used without any adjustment (shift and tilt) to compute NAVD 88(2007) elevations from GPS-measured 2007 NAD 83 ellipsoid heights.

To compute NAVD 88(1991) elevations from 2007 NAD 83 ellipsoid heights, a single shift of -0.103 ft (shown to 3 decimal places only to avoid rounding errors when comparing computations) should be applied to GEOID09. A negative shift makes geoid separations (N) in the Madison area less negative; when the adjusted (less negative) geoid separation (N) is subtracted from the ellipsoid height (h), a smaller (less positive) NAVD 88 elevation will result. This is what we expect: NAVD 88(1991) elevations are smaller than NAVD 88(2007) elevations, meaning the NAVD 88(1991) datum is higher than the NAVD 88(2007) datum (Figure 02).

As modified, GEOID09 can generally reproduce published NAVD 88(1991) or NAVD 88(2007) elevations at the 34 stations tested to within +/- 0.05 feet (80% to +/- 0.03 ft), which is comparable to the accuracy limits of RTK GPS.

Table 14: *Recommended* GEOID09 Shift and Tilt Parameters (Derived from Table 13)
Approximate Dane County Coordinates, NAD 83 Datum, of Emil Street base are N=469050 US ft, E=814396 US ft

Parameter	2007 NAD 83, NAVD 88(1991) (Table 12, Figure 19)	2007 NAD 83, NAVD 88(2007) (Table 11, Figure 18)	1997 NAD 83, NAVD 88(1991)
Shift at Emil Street Base	-0.103ft	*none*	*not computed*
East-West Tilt	*None*	*none*	*not computed*
North-South Tilt	*None*	*none*	*not computed*

3.10. COMPARING GEOID03 AND GEOID09

Figure 20 below shows a comparison of the NAVD 88(2007) - 2007 NAD 83 adjusted GEOID03 (Table 05 / Figure 14 & Figure 15) and the unadjusted GEOID09. Both are designed to compute NAVD 88(2007) elevations from 2007 NAD 83 ellipsoid heights. Geoid separation values were generated from each using a 1-minute grid (0°01' latitude, 0°01'longitude), and 0.01-foot contours were interpolated from the difference values (GEOID09 minus adjusted GEOID03).

The contours show the change in *curvature* of GEOID09 from GEOID03, resulting from the updated gravity data and bench mark data used for GEOID09. The largest change, reaching +0.06 ft, is still within the general accuracy range of RTK GPS, but it unfortunately is centered near Lake Mendota and affects central and east Madison.

Where the difference contours are *positive* (i.e. State Capitol approx. +0.05 ft), the GEOID09 geoid separation is less negative than that from the adjusted GEOID03. When the (negative) geoid separation is subtracted from GPS-measured ellipsoid height, the resulting elevation is *less positive* (lower) using GEOID09 than using the adjusted GEOID03.

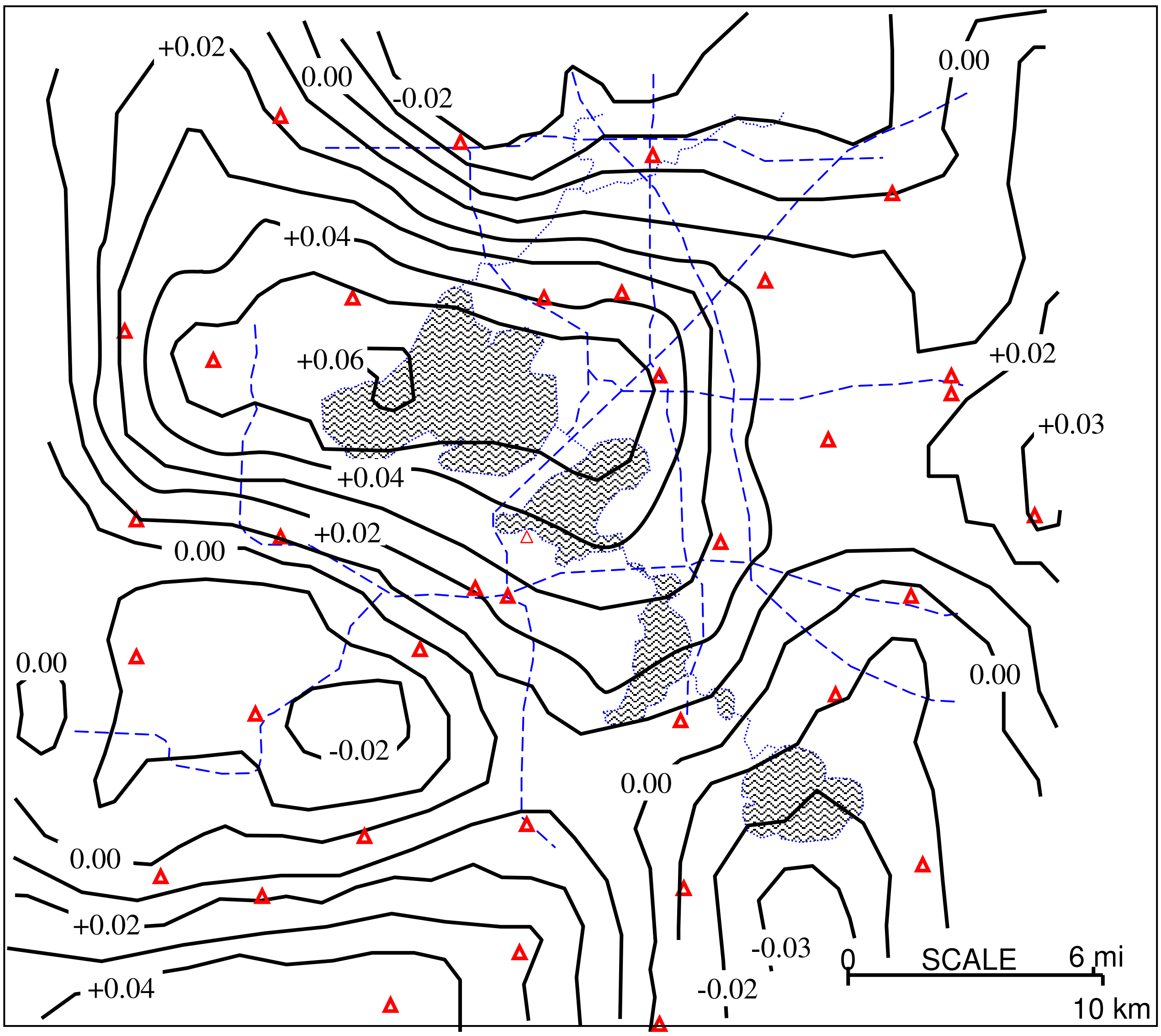
Sample computation (inside +0.06 ft contour)

GEOID03 adjustment parameters are from Table 09 (2007 NAD 83, NAVD 88(2007))
Position = 43°07'00.0"N, 89°27'00.0"W (Dane County Coordinates N=498131.37, E=803582.61 US ft)
Unadjusted GEOID03 geoid separation = -34.344m = -112.677 ft

Sample Point:	N = 498131 US ft	E = 803583 US ft
Origin (Emil St):	N = 469050 US ft	E = 814396 US ft
Origin to Sample:	dN = +29081 ft	dE = –10813 ft
2007-2007 tilt values:	tiltN = –0.83 ppm	tiltE = +1.58 ppm

North tilt component of geoid residual (Origin to Sample) = (dN x tiltN) = (+29081 x –0.83/1000000) = –0.024 ft
East tilt component of geoid residual (Origin to Sample) = (dE x tiltE) = (–10813 x +1.58/1000000) = –0.017 ft
Shift component of geoid residual at Sayle St = +0.027 ft
Geoid residual at Emil St = sum of 3 components = (–0.024 –0.017 +0.027) = –0.014 ft
Adjusted 2007-2007 GEOID03 geoid sep = (GEOID03 – geoid residual) = (–112.677 – (–0.014)) = –112.663 ft
Unadjusted GEOID09 geoid separation = -34.321m = –112.601 ft
Difference (GEOID09 minus adjusted GEOID03) = –112.601 ft –(–112.663 ft) = +0.062 ft = +0.06 ft rounded

Figure 20: GEOID09 vs. Adjusted GEOID03 (feet). Difference contours are unadjusted GEOID09 geoid separation minus NAVD 88(2007) – 2007 NAD 83 adjusted GEOID03 geoid separation (Table 05 / Figure 14)



3.11. GEOID12A:

In 2012 the NGS released the GEOID12A geoid model, originally named GEOID12 but then corrected as GEOID12A. GEOID12A is designed to convert NAD 83(2011)(Epoch 2010.00) ellipsoid heights to NAVD 88(2012) elevations. These ellipsoid heights and elevations were obtained for the previously analyzed 34 stations from NGS data sheets retrieved Jan 11, 2014 (same as Aug. 14,2012 sheets, except non-leveled NAVD 88(2012) elevations weren't available then). Table 15 shows a comparison.

Table 15: GEOID12A Geoid Residuals for NAVD 88(2012) Elevations and NAD 83(2011)(Epoch 2010.00) Ellipsoid Heights From NGS Data Sheets retrieved Jan. 11, 2014

Station	H NAVD 88(2012), ft	NAVD 88 source	h NAD 83(2011)(Epoch 2010.00) ellip ht, ft	Ellip Ht 95% accuracy, ft	N GEOID12A, ft	h-H, ft	ΔN = N-(h-H), ft
MADISON S GPS	868.07	Leveled	755.57	0.017	-112.48	-112.50	0.02
BURKE S GPS	851.32	Leveled	738.32	0.019	-113.04	-113.00	-0.04
OREGON GPS	977.98	Leveled	865.89	0.013	-112.07	-112.09	0.02
FITCHBURG S GPS	1059.80	Leveled	947.84	0.016	-111.95	-111.96	0.01
RUTLAND N GPS	920.49	Leveled	808.32	0.014	-112.16	-112.17	0.01
COTTAGE GROVE S GPS	930.90	Leveled	818.04	0.011	-112.84	-112.86	0.02
BURKE N GPS	884.20	Leveled	770.54	0.011	-113.68	-113.66	-0.02
WESTPORT N GPS	937.99	Leveled	824.55	0.012	-113.48	-113.44	-0.04
OREGON E GPS	1034.21	Leveled	922.40	0.015	-111.81	-111.81	0.00
87Y A	901.11	Leveled	787.80	0.024	-113.28	-113.31	0.03
VERONA GPS	952.86	Leveled	841.08	0.014	-111.77	-111.78	0.01
COTTAGE GROVE GPS	901.57	Leveled	788.22	0.010	-113.32	-113.35	0.03
ROCK	1059.66	Leveled	946.38	0.010	-113.29	-113.28	-0.01
SUN PRAIRIE W GPS	961.22	Leveled	847.43	0.012	-113.77	-113.79	0.02
STOUGHTON GPS	888.88	Leveled	776.50	0.015	-112.40	-112.38	-0.02
FITCHBURG N GPS	936.45	GPS-derived	824.10	0.027	-112.33	-112.35	0.02
MONONA GPS	863.22	GPS-derived	750.43	0.013	-112.79	-112.79	0.00
MADISON GPS	893.40	GPS-derived	780.32	0.015	-113.09	-113.08	-0.01
DUNN C GPS	858.36	GPS-derived	745.90	0.024	-112.46	-112.46	0.00
MIDDLETON GPS	1080.02	GPS-derived	967.66	0.022	-112.36	-112.36	0.00
ARP 2 MSN	***855.38***	GPS-derived	742.23	0.053	-113.22	-113.15	***-0.07***
WESTPORT S GPS	872.50	GPS-derived	759.62	0.015	-112.88	-112.88	0.00
COTTAGE GROVE W GPS	891.96	GPS-derived	778.89	0.013	-113.07	-113.07	0.00
VERONA E GPS	953.51	GPS-derived	841.37	0.019	-112.15	-112.14	-0.01
BURKE E GPS	925.56	GPS-derived	812.18	0.016	-113.39	-113.38	-0.01
PLEASANT SPRINGS N GPS	861.42	GPS-derived	748.75	0.011	-112.66	-112.67	0.01
MOREPORT AZ MK	926.05	GPS-derived	813.43	0.019	-112.59	-112.62	0.03
CROSS PLAINS E GPS	1077.52	GPS-derived	965.24	0.013	-112.27	-112.28	0.01
VERONA N GPS	1007.25	GPS-derived	895.11	0.013	-112.14	-112.14	0.00
SPRINGFIELD S GPS	1058.23	GPS-derived	945.66	0.011	-112.57	-112.57	0.00
OREGON C GPS	918.24	GPS-derived	806.60	0.014	-111.64	-111.64	0.00
MONTROSE N GPS	927.29	GPS-derived	815.51	0.013	-111.76	-111.78	0.02
RUTLAND C GPS	915.09	GPS-derived	803.16	0.018	-111.91	-111.93	0.02
COTTAGE GROVE E GPS	884.28	GPS-derived	771.22	0.011	-113.04	-113.06	0.02

The NAVD 88(2012) – NAD 83(2011)(Epoch 2010.00) geoid residuals using GEOID12A for these 33 of the 34 stations (except ARP 2 MSN) lie between -0.04ft and +0.03ft, with an average of +0.004ft. They are centered around zero, confirming that the NGS used NAVD 88(2012) elevations to help calibrate GEOID12A. ARP 2 MSN is an outlier, mostly because its NAVD 88 elevation is unchanged from the 1991 value. It also has lower accuracy and questionable vertical stability.

Table 16 shows NAVD 88(1991) –NAD 83(2011)(Epoch 2010.00) geoid residuals:

Table 16: GEOID12A Geoid Residuals for NAVD 88(1991) Elevations and NAD 83(2011)(Epoch 2010.00) Ellipsoid Heights From NGS Data Sheets retrieved August 14, 2012

Station	H NAVD 88(1991), ft	NAVD 88 source	NAVD 88 Date	h NAD 83(2011)(Epoch 2010.00) ellip ht, ft	N GEOID12A, ft	h-H, ft	ΔN = N-(h-H), ft
MADISON S GPS	867.91	Leveling	2/25/2004	755.57	-112.48	-112.34	-0.14
BURKE S GPS	851.16	Leveling	2/25/2004	738.32	-113.04	-112.84	-0.20
OREGON GPS	977.81	Leveling	2/25/2004	865.89	-112.07	-111.92	-0.15
FITCHBURG S GPS	1059.61	Leveling	2/25/2004	947.84	-111.95	-111.77	-0.18
RUTLAND N GPS	920.33	Leveling	2/25/2004	808.32	-112.16	-112.01	-0.15
COTTAGE GROVE S GPS	930.73	Leveling	2/25/2004	818.04	-112.84	-112.69	-0.15
BURKE N GPS	884.02	Leveling	2/25/2004	770.54	-113.68	-113.48	-0.20
WESTPORT N GPS	937.82	Leveling	2/25/2004	824.55	-113.48	-113.27	-0.21
OREGON E GPS	1034.06	Leveling	2/25/2004	922.40	-111.81	-111.66	-0.15
87Y A	900.92	Leveling	2/25/2004	787.80	-113.28	-113.12	-0.16
VERONA GPS	952.66	Leveling	2/25/2004	841.08	-111.77	-111.58	-0.19
COTTAGE GROVE GPS	901.38	Leveling	2/25/2004	788.22	-113.32	-113.16	-0.16
ROCK	1059.48	Leveling	2/25/2004	946.38	-113.29	-113.10	-0.19
SUN PRAIRIE W GPS	961.02	Leveling	2/25/2004	847.43	-113.77	-113.59	-0.18
STOUGHTON GPS	888.72	Leveling	2/25/2004	776.50	-112.40	-112.22	-0.18
FITCHBURG N GPS	936.25	GPS-derived	4/2/2004	824.10	-112.33	-112.15	-0.18
MONONA GPS	863.09	GPS-derived	4/2/2004	750.43	-112.79	-112.66	-0.13
MADISON GPS	893.24	GPS-derived	4/2/2004	780.32	-113.09	-112.92	-0.17
DUNN C GPS	858.17	GPS-derived	4/2/2004	745.90	-112.46	-112.27	-0.19
MIDDLETON GPS	1079.89	GPS-derived	4/2/2004	967.66	-112.36	-112.23	-0.13
ARP 2 MSN	***855.38***	GPS-derived	4/28/1999	742.23	-113.22	-113.15	**-0.07**
WESTPORT S GPS	872.34	GPS-derived	4/2/2004	759.62	-112.88	-112.72	-0.16
COTTAGE GROVE W GPS	891.80	GPS-derived	4/2/2004	778.89	-113.07	-112.91	-0.16
VERONA E GPS	953.31	GPS-derived	4/2/2004	841.37	-112.15	-111.94	-0.21
BURKE E GPS	925.39	GPS-derived	4/2/2004	812.18	-113.39	-113.21	-0.18
PLEASANT SPRINGS N GPS	861.22	GPS-derived	4/2/2004	748.75	-112.66	-112.47	-0.19
MOREPORT AZ MK	925.88	GPS-derived	9/30/2002	813.43	-112.59	-112.45	-0.14
CROSS PLAINS E GPS	1077.39	GPS-derived	4/2/2004	965.24	-112.27	-112.15	-0.12
VERONA N GPS	1007.05	GPS-derived	4/2/2004	895.11	-112.14	-111.94	-0.20
SPRINGFIELD S GPS	1058.13	GPS-derived	4/2/2004	945.66	-112.57	-112.47	-0.10
OREGON C GPS	918.08	GPS-derived	4/2/2004	806.60	-111.64	-111.48	-0.16
MONTROSE N GPS	927.07	GPS-derived	4/2/2004	815.51	-111.76	-111.56	-0.20
RUTLAND C GPS	914.89	GPS-derived	4/2/2004	803.16	-111.91	-111.73	-0.18
COTTAGE GROVE E GPS	884.09	GPS-derived	4/2/2004	771.22	-113.04	-112.87	-0.17

The NAVD 88(1991) – NAD 83(2011)(Epoch 2010.00) geoid residuals (ΔN) using GEOID12A for these 34 stations lie between -0.21ft and -0.10ft (ARP 2 MSN -0.07ft), averaging -0.17 ft (including or excluding ARP 2 MSN). Again, Station ARP 2 MSN (PID #OM1387) is an outlier, as in Table 07, indicating that the station appears to have sunk after its NAVD 88(1991) elevation was computed in 1999. The average geoid residual of –0.17 ft is equal to the average shift between NAVD 88(1991) and NAVD 88(2012) in the Madison area (Figure 02), confirming that GEOID12A is designed to produce NAVD 88(2012) elevations.

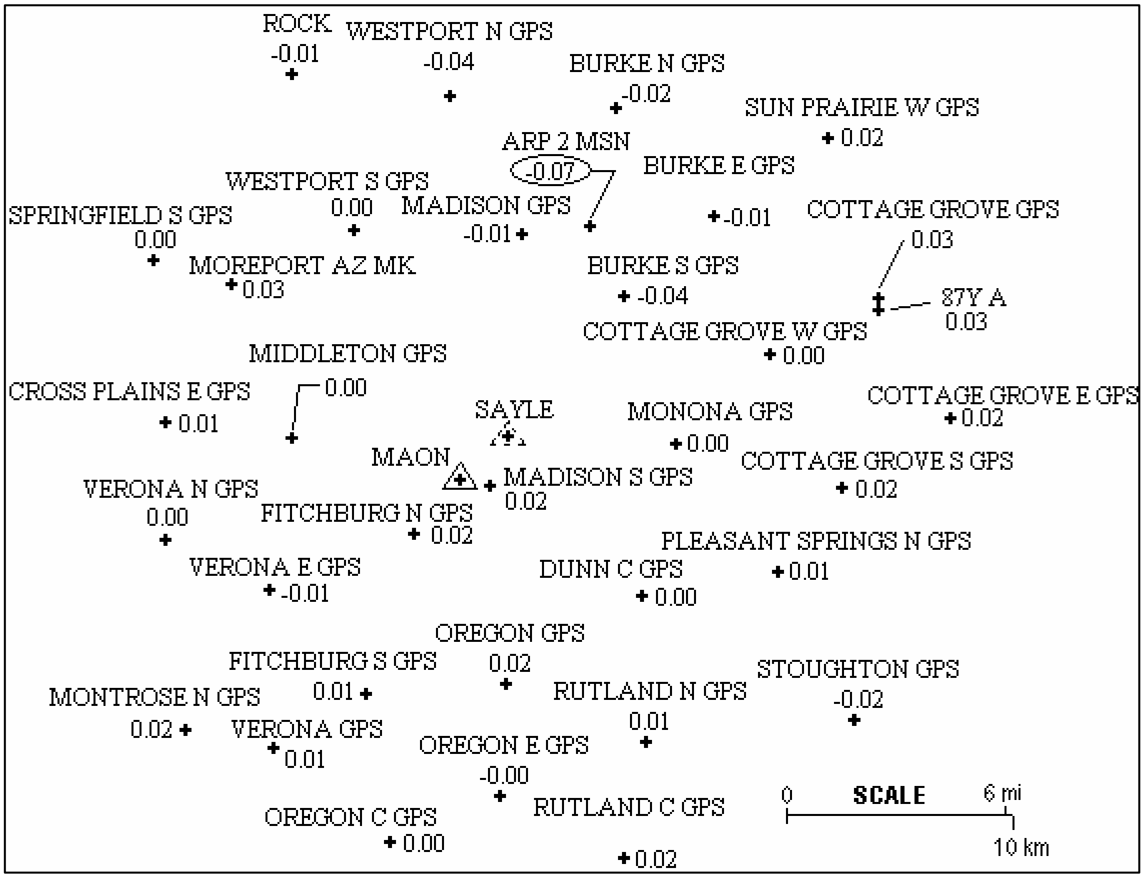


Figure 21: 2011 / 2012 GEOID12A Residuals, feet
Using NAD 83(2011)(Epoch 2010.00) ellipsoid heights and NAVD 88(2012) elevations

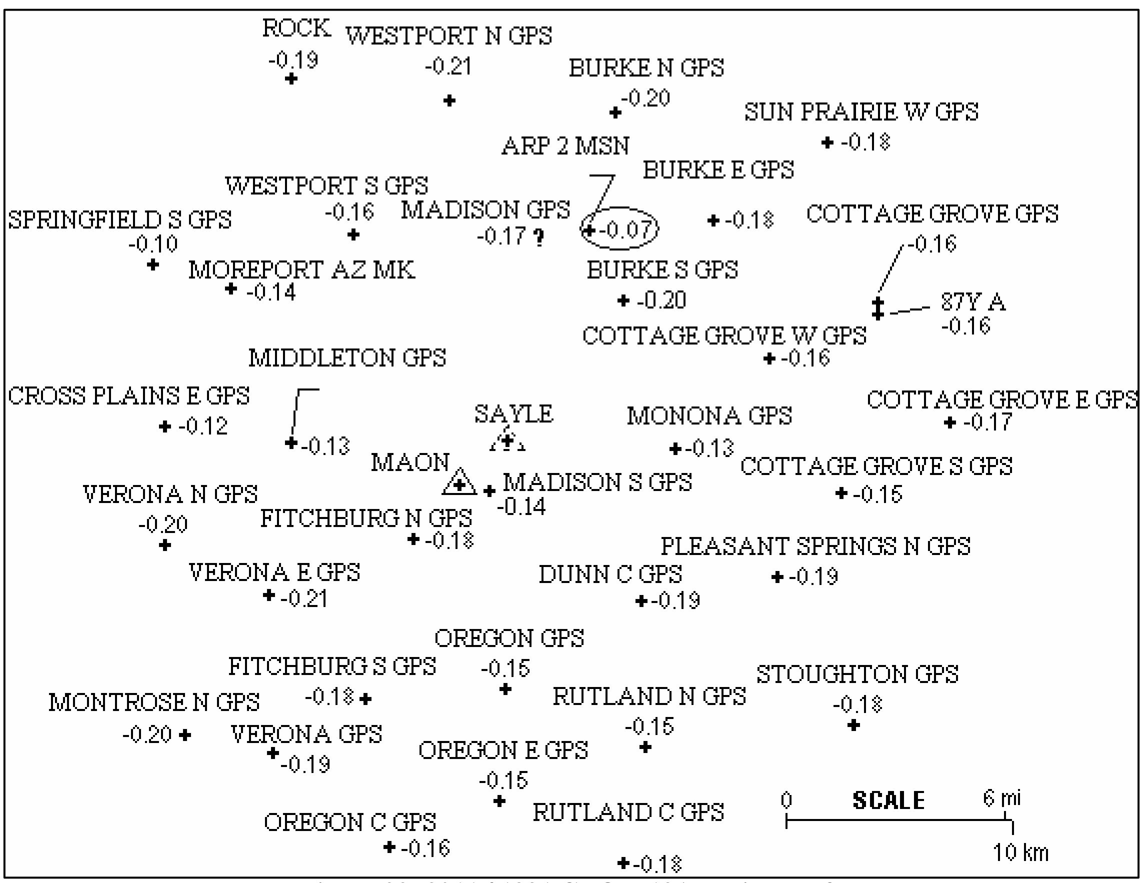


Figure 22: 2011 / 1991 GEOID12A Residuals, feet
Using NAD 83(2011)(Epoch 2010.00) ellipsoid heights and NAVD 88(1991) elevations

Table 17 shows the best-fit shift and tilt parameters to minimize the GEOID12A geoid residuals:

Table 17: GEOID12A Shift and Tilt Parameters
Approximate Dane County Coordinates, NAD 83 Datum, of Emil Street base are N=469050 US ft, E=814396 US ft
Values in parentheses are standard deviations from least-squares best-fit computation of shift & tilt parameters.

NOTE: The Sep. 24 2012 & Mar. 23 2013 versions had the east-west and north-south tilts accidentally switched. Also those versions used only leveled station data for the NAD 83(2011)(Epoch 2010.00) / NAVD 88(2012) analysis.

Parameter	NAD 83(2011)(Epoch 2010.00), NAVD 88(1991) (Table 16, Figure 22)	NAD 83(2011)(Epoch 2010.00), NAVD 88(2012) (Table 15, Figure 21)
Shift at Emil Street Base	-0.167ft (+/-0.005ft)	+0.004ft (+/-0.003ft)
East-West Tilt	-0.06ppm (+/-0.14ppm)	+0.09ppm (+/-0.08ppm)
North-South Tilt	-0.03ppm (+/-0.15ppm)	-0.15ppm (+/-0.08ppm)

The tilts are similar in magnitude to their standard deviations, indicating no clear improvement by applying tilt corrections to GEOID12A for either data set. 0.1 ppm is 0.01 foot in 19 miles, which is an insignificant correction for all but the most precise GPS techniques.

The shift value for the NAD 83(2011)(Epoch 2010.00) - NAVD 88(2012) data is an insignificant 0.004 ft. The shift for the NAD 83(2011)(Epoch 2010.00) - NAVD 88(1991) data is –0.17 ft rounded, matching the average shift between NAVD 88(1991) and NAVD 88(2012) adjustments in the Madison area (Figure 02).

Adjusted geoid residuals (after applying shift and tilts) for 33 of the 34 stations (excluding ARP 2 MSN) still reach magnitudes of 0.05 ft using NAVD 88(2012) and 0.06 ft using NAVD 88(1991), a reminder that some amount of random error in NGS-published ellipsoid heights, NAVD 88 elevations and/or geoid models is present. The outlier station ARP 2 MSN has a much larger adjusted geoid residual, +0.07 ft using NAVD 88(2012) and -0.10 ft using NAVD 88(1991), but it does not significantly affect the shift and tilt computations because each station is weighted by the NGS's published ellipsoid height 95% accuracy. ARP 2 MSN's published accuracy is about 5 times worse than average.

GEOID12A Results: Thus, in the Madison area, to compute elevations consistent with RTK GPS accuracy from the Madison base station, GEOID12A can be used without any adjustment (shift and tilt) to compute NAVD 88(2012) elevations from GPS-measured NAD(2011)(Epoch 2010.00) ellipsoid heights.

To compute NAVD 88(1991) elevations from NAD(2011)(Epoch 2010.00) ellipsoid heights, a single shift of -0.167 ft (shown to 3 decimal places only to avoid rounding errors when comparing computations) should be applied to GEOID12A. A negative shift makes geoid separations (N) in the Madison area less negative; when the adjusted (less negative) geoid separation (N) is subtracted from the ellipsoid height (h), a smaller (less positive) NAVD 88 elevation will result. This is what we expect: NAVD 88(1991) elevations are smaller than NAVD 88(2012) elevations, meaning the NAVD 88(1991) datum is higher than the NAVD 88(2012) datum (Figure 02).

As modified, GEOID12A can generally reproduce published NAVD 88 elevations at the 34 stations tested to within +/- 0.05 feet (91% to +/- 0.03 ft) for NAVD 88(2012), and to within +/- 0.06 ft (94% to +/- 0.04 ft, 71% to +/-0.03 ft) for NAVD 88(1991), ranges which are comparable to the accuracy limits of RTK GPS.

Table 18: *Recommended* GEOID12A Shift and Tilt Parameters (Derived from Table 17)
Approximate Dane County Coordinates, NAD 83 Datum, of Emil Street base are N=469050 US ft, E=814396 US ft
(Tilt origin coordinates not necessary if no tilt parameters used)

Parameter	NAD 83(2011)(Epoch 2010.00), NAVD 88(1991) (Table 16, Figure 22)	NAD 83(2011)(Epoch 2010.00), NAVD 88(2012) (Table 15, Figure 21)
Shift at Emil Street Base	-0.167ft	*none*
East-West Tilt	*None*	*none*
North-South Tilt	*None*	*none*

3.12. COMPARING GEOID12A AND GEOID03

Comparing GEOID12A to previous geoid models is difficult because it is based on new adjustments of both NAD 83 and NAVD 88. Since the Emil Street base station (MAON) was established in 2009, the City of Madison has been using GEOID03 adjusted to fit 2007 NAD 83 ellipsoid heights and NAVD 88(1991) elevations (Table 07 / Figure 17), so a comparison will be made to that.

Ellipsoid heights changed an average of -0.10 ft (Table 03) from 2007 NAD 83 to NAD 83(2011)(Epoch 2010.00), and orthometric elevations changed +0.17 ft (Figure 02) from NAVD 88(1991) to NAVD 88(2012). So +0.27 feet will be added to GEOID12A geoid separations (making them less negative) to directly compare them to the adjusted GEOID03 values described above. Figure 23 illustrates this compensation.Geoid separation values were generated from each using a 1-minute grid (0°01' latitude, 0°01'longitude), and 0.01-foot contours were interpolated from the difference values (= GEOID12A + 0.27 ft - modified GEOID03).

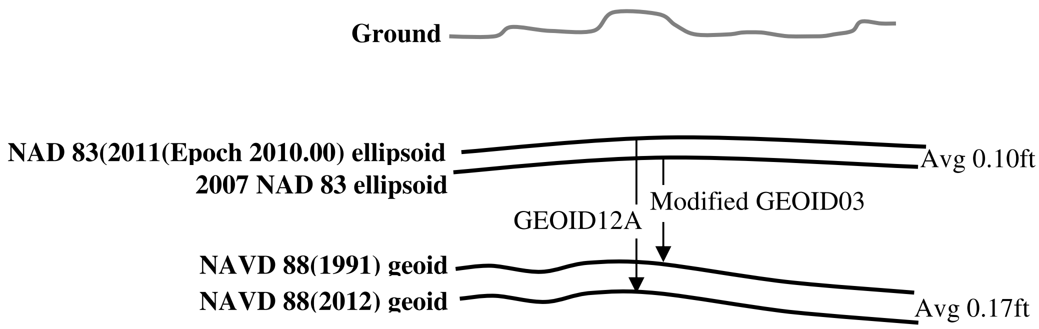
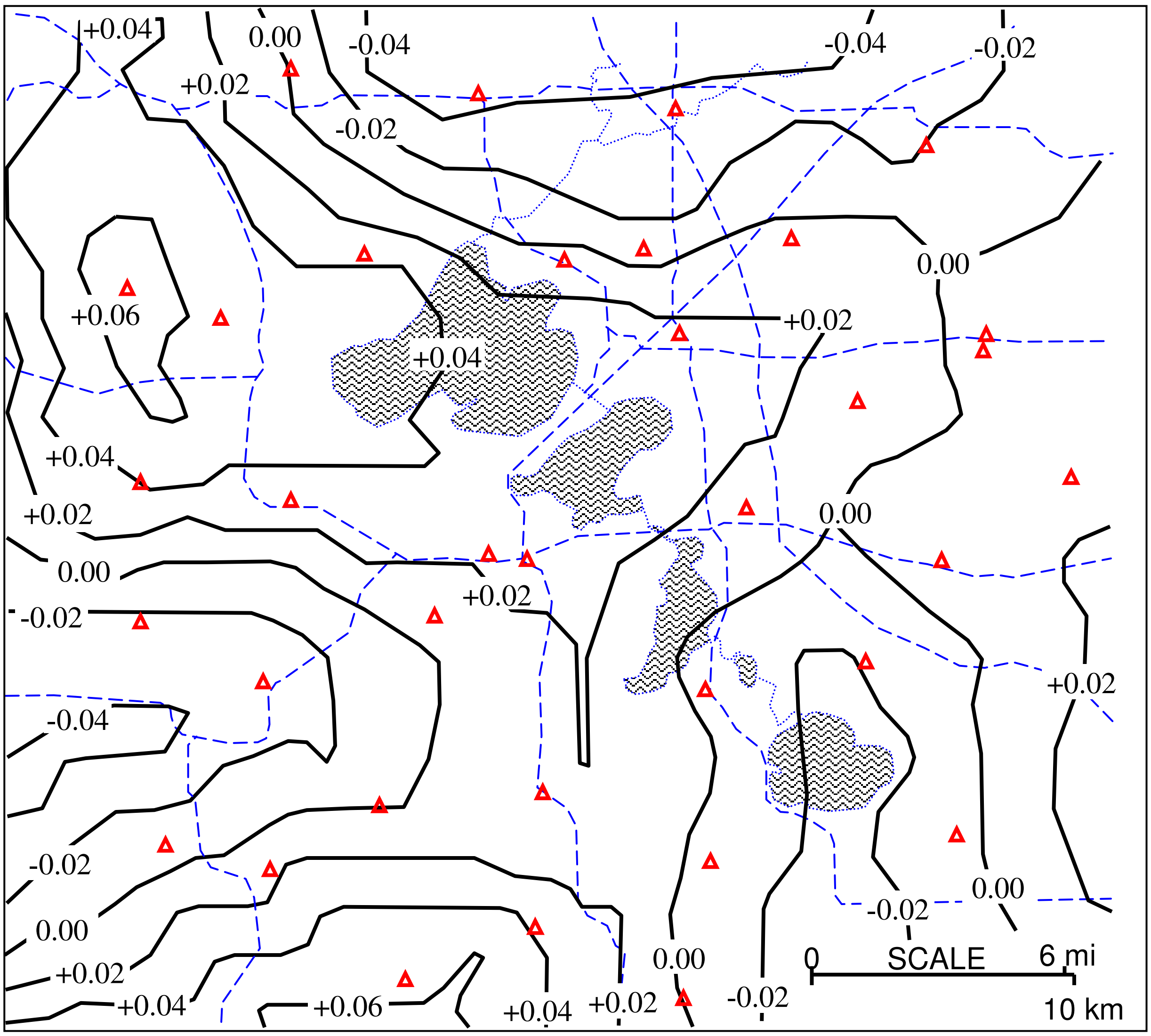


Figure 23: Adjustments to Compare GEOID12A & Adjusted GEOID03

The contours show the change in *curvature* of GEOID12A from GEOID03, resulting from the updated gravity data and bench mark data used for GEOID12A. The largest difference of +0.07 ft,near station SPRINGFIELD S GPS west of Madison, is still within the general accuracy range of RTK GPS. Changes in Madison are within 0.05 feet. This shows that for RTK GPS, using GEOID12A (with a -0.167 ft shift per Table 14) to convert NAD 83(2011)(Epoch 2010.00) ellipsoid heights to NAVD 88(1991) elevations will not result in significantly different NAVD 88(1991) elevations than what the City of Madison has been using previously.

Figure 24: GEOID12A vs. Adjusted GEOID03 (feet). Difference contours are unadjusted GEOID12A geoid separation, plus 0.27 feet, minus NAVD 88(1991) – 2007 NAD 83 adjusted GEOID03 geoid separation (Table 6 / Figure 16 & Figure 17)



3.13. GEOID / ELLIPSOID SUMMARY SO FAR: The purpose of the NAD 83 ellipsoid / NAVD 88 geoid comparisons above is to be able to convert GPS-measured ellipsoid heights to NAVD 88 elevations. Before getting into GPS measurement details in the next sections, here is a summary of key points so far.

- GPS measures height relative to an ellipsoid (e.g., NAD 83), and traditional instruments (levels, total stations) measure elevation relative to a geoid (e.g., NAVD 88).
- The relation between the geoid and ellipsoid is not exactly known. It can be estimated by a geoid model, such as the NGS's GEOID03, GEOID09 or GEOID12A.
- The NAD 83 ellipsoid and NAVD 88 geoid each have different adjustments, with different height / elevation values for the same stations. The difference could be from actual ground movement relative to datum, and/or movement of the datum relative to ground (i.e. correction of measurements and computations used to establish the previous datum). In the Madison area, average changes in NAD 83 ellipsoid height were -0.01 ft from 1997 to 2007, and -0.10 ft from 2007 to 2011(Epoch 2010.00) (Table 03 & Figure 08). Average changes in NAVD 88 elevation were +0.11 feet from the original adjustment (1991) to 2007, and +0.06 ft from 2007 to 2012 (Figure 02).
- The GEOID03 geoid model, without adjustment, predicts geoid separations (N = h-H) that agree with published 2007 NAD 83 ellipsoid heights (h) and NAVD 88(1991) elevations (H) to within 0.19 feet in the Madison area (Table 07). GEOID03 can be shifted vertically and tilted to better agree with published ellipsoid heights and NAVD 88 elevations, to within 0.05 feet (majority within 0.03 feet) (Figure 17). The City of Madison Engineering and Parks Divisions used this adjustment since 2009 for RTK GPS, to produce NAVD 88(1991) elevations from the Emil Street GPS base station or from WISCORS.
- The GEOID09 geoid model, without adjustment, predicts geoid separations (N = h-H) that agree with published 2007 NAD 83 ellipsoid heights (h) and NAVD 88(2007) elevations (H) to within 0.06 feet (majority within 0.03 feet) in the Madison area (Table 11). Shifting and tilting GEOID09 does not produce significant improvement, but a shift of –0.10 ft is necessary to predict NAVD 88(1991) elevations as accurately (Table 13). Differences between GEOID09 and GEOID03 vary by location because of the different (and assumedly improved) curvature of GEOID09. However, GEOID09 was never used by the City of Madison Engineering and Parks Divisions because of insignificant differences from the adjusted GEOID03 for RTK GPS.
- The GEOID12A geoid model, without adjustment, predicts geoid separations (N = h-H) that agree with published NAD 83(2011)(Epoch 2010.00) ellipsoid heights (h) and NAVD 88(2012) elevations (H) to within 0.04 feet (majority within 0.02 feet) in the Madison area (Table 16). Shifting and tilting GEOID12A does not produce significant improvement, but a shift of –0.167 ft is necessary to convert NAD 83(2011)(Epoch 2010.00) ellipsoid heights to NAVD 88(1991) elevations as accurately (Table 18). When comparing GEOID12A to the adjusted GEOID03 the City of Madison used previously, the change in ellipsoid heights (average -0.10 ft) from 2007 NAD 83 to NAD 83(2011)(Epoch 2010.00) must be considered. After compensating (Figure 23 & Figure 24), the adjusted GEOID12A geoid model produces NAVD 88(1991) elevations within 0.05 ft of the same produced by the adjusted GEOID03 geoid model, with differences varying by location because of the different (and assumedly improved) curvature GEOID12A.
- Just because a geoid model can be adjusted to match an ellipsoid and a geoid to a few hundredths of a foot, doesn't mean that a particular GPS technique can produce elevations that accurate. More below.
- **ALL OF THE ABOVE MAY NOT MATTER** if the only goal is elevations consistent with a particular previous project, right or wrong. In that case one needs to check into bench marks from that project, with published elevations from that project.

4.1. GPS-DERIVED ELEVATIONS This is not a GPS textbook; complete background material is not presented here. Much of the information is based on the old Sayle Street City of Madison GPS base station, which was moved to Emil Street in 2009. However, the computations using Sayle Street are instructive, particularly in how its old broadcast ellipsoid height was computed from a NAVD 88 elevation and geoid model, rather than a true ellipsoid height.

Figure 25 and Table 19 shows conceptual formula and profile view for computing the NAVD 88 (geoid) elevation of Station 2 from the given NAVD 88 elevation of station 1 using GPS.

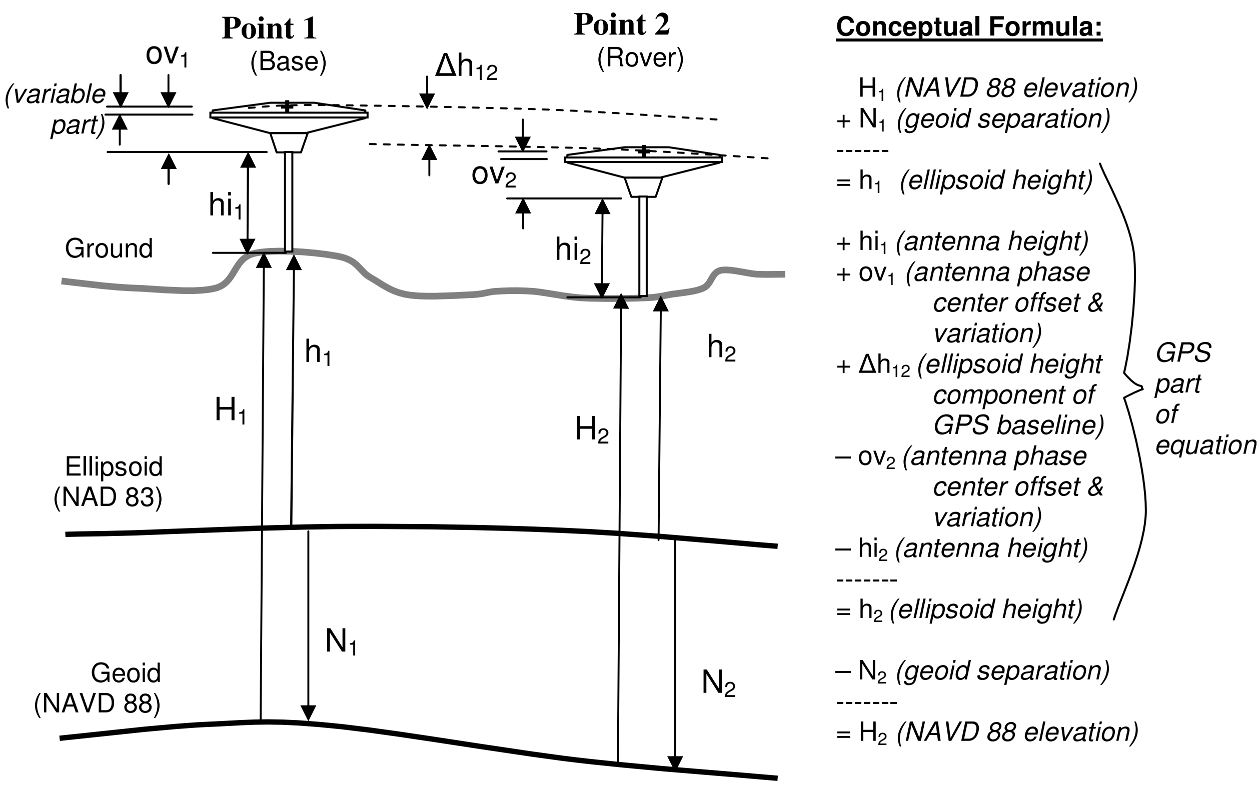


Figure 25: Conceptual GPS Elevation Computation

Table 19: Details of GPS-Derived NAVD 88 Elevation Computation (see Figure 25)

Parameter	***Discussion***
H_1	Elevation of Point 1 above the geoid (NAVD 88)
N_1	Geoid separation at Point 1 (N is negative). This could be estimated from a geoid model, such as GEOID03, or better information.
h_1	Ellipsoid height of Point 1. If point 1 has a published ellipsoid height, then N_1 and H_1 are not needed. Correct processing of the GPS baseline requires an ellipsoid height at Point 1. If one incorrectly assumes the ellipsoid and geoid are the same datum, this will introduce a few parts per million of error in the GPS baseline (Δh_{12}) alone, in addition to the ~0.1 foot per mile slope of the geoid relative to the ellipsoid in this area (Figure 06).
hi_1	Antenna height at Point 1, from ground station to bottom of antenna mount
ov_1	Antenna phase center offset and variation, from bottom of antenna mount up to antenna phase center. The phase center for the "L1" carrier signal is usually given. The GPS baseline (Δh_{12}) is measured from phase center to phase center. The phase center offset consists of a constant offset (o) and a variable offset (v), because the phase center changes with satellite elevation angle. These parameters are taken into account by specifying the antenna type (antenna model) in the GPS controller. Assuming a single antenna height from the ground point to the phase center, without specifying the antenna type, disregards the variable portion of the offset.
Δh_{12}	The ellipsoid height component of the GPS baseline. This is the fundamental GPS-measured value, and <u>often the weakest link</u>. Even if you know N_1 and N_2 very precisely to relate h's and H's (using a tilted & shifted GEOID03 geoid model, for example), the accuracy of H_2 is still only as good as Δh_{12}. Typical RTK GPS vertical accuracy for a 5-second observation is about +/-(0.07 ft + 1 ppm) at ~68% confidence, or about double that +/-(0.14 ft + 2 ppm) for 95% confidence. At 6 miles from the base station, 95% confidence would be about +/-(0.14 ft + 0.06 ft) = +/- 0.20 ft. That's about as large as the unmodified GEOID03 geoid residuals in the Madison area! Extending RTK observation time to a few minutes improves accuracy somewhat, but some random errors only change over longer time periods, as satellite orbits and atmospheric conditions change. For all GPS methods, accuracy can only be reliably improved by averaging observations from different times of day (at least 1 hour apart, to allow satellite orbits to change), and ideally on different days (to allow atmospheric conditions to change). <u>Most importantly, multipath reflection errors or bad initialization (solution of integer ambiguities) may cause much larger errors</u>. A less significant issue is that GPS baselines are technically solved in the satellite datum, the World Geodetic System of 1984 (WGS 84), which is considered virtually identical to the International Terrestrial Reference Frame of 2000 (ITRF00). Relative to WGS 84, the NAD 83 ellipsoid is shifted about 2 meters, rotated about 0.1 part per million, and even scaled about 1 part per billion (see www.ngs.noaa.gov/TOOLS/Htdp/Htdp.shtml). This means that the ellipsoid height component of a baseline (Δh_{12}) should technically be transformed from WGS 84 to NAD 83. However, the effect is basically undetectable for the short (< 20-30 km) baselines dealt with here. Similarly, for these short baselines, the NAD 83 coordinates for the Madison base station can be entered as though they were WGS 84 coordinates, and the rover position will be in NAD 83, even though the processing software assumes the WGS 84 datum is being used.
ov_2	See ov_1. If the same antenna type is used at Points 1 and 2, then ov_1 and ov_2 effectively cancel each other for shorter baselines. However, if the Point 1 antenna (e.g. base station) is different than the Point 2 antenna (e.g. your rover), then both antenna types need to be specified to account for the difference.
hi_2	Antenna height at Point 2, from ground station to bottom of antenna mount.
h_2	Computed ellipsoid height at Point 2.
N_2	Geoid separation at Point 2 (N is negative). N_2 must be consistent with N_1. They must be from the same geoid model, like GEOID03, and the geoid model must accurately model the relative separation between the ellipsoid and geoid (NAVD 88) surfaces. A shifted and tilted GEOID03, as computed in previous sections, would keep errors in the *difference* between N_1 & N_2 to within about 0.05 feet (mostly within 0.03 feet) in the Madison area.
H_2	NAVD 88 elevation at Point 2. If a good geoid model is used, and the antenna heights and antenna types are specified, the largest error in H_2 is probably from the GPS baseline (Δh_{12}).

4.2. OLD MADISON GPS BASE STATION (SAYLE STREET): Table 20 lists city-published vertical values determined circa 2005 for the old City of Madison GPS Base Station at 1120 Sayle Street in central Madison. See Appendix 2 at end for additional information, and see http://gis.cityofmadison.com/Madison_GPS/ for details and updates. As of November 2007, the base is not an NGS-published station. Note that the geoid residual (ΔN) is calculated in Table 20, and shown two different ways in Figure 26. The base station's observed geoid residual of –0.11 feet is similar to geoid residuals at nearby NGS stations (Figure 10), and similar to the best-fit GEOID03 geoid model shift of –0.09 ft, which is computed at the base station (Figure 13).

Table 20: Madison GPS Base Station –Vertical Values Determined Circa 2005

Parameter	***Value & Notes***
H	**875.69 ft** (266.911 m), NAVD 88(1991) datum. Determined in 2005 by leveling from nearby NGS station 2V02, which had a published NAVD 88 elevation of 854.35ft (260.406m) in 2004.
h	**763.38 ft** (232.679 m), 1997 NAD 83 ellipsoid height. GPS-measured from 1999 NAD 83 ellipsoid heights at nearby NGS stations ROCK, KOLLATH and COTTAGE GROVE GPS (UW-Madison project). Note that per Figure 08, nearby NGS stations show almost no change from 1997 to 2007.
h-H	-**112.31 ft** (-34.232 m). Difference of NAD 83 ellipsoid height and NAVD 88 elevation. This is not necessarily the "true" geoid separation, since both the h and H values may have measurement error.
N	**-112.42 ft** (-34.266 m) per GEOID03 geoid model. Notice that N does not equal (h-H).
ΔN =N-(h-H)	ΔN = N-(h-H) = (-112.42)-(-112.31 ft) =**-0.11 ft** (-0.034 m). This is the geoid residual, which can be visualized as creating a second geoid or a second ellipsoid (Figure 26). This compares well with the GEOID03 best-fit shift parameter of –0.09 ft between NAVD 88(1991) and either 1997 or 2007 NAD 83 (Table 08).
h' =H+N	Using the second ellipsoid concept in Figure 26, the second ellipsoid height (h') is 875.69 + (-112.42) = **763.27 ft** (232.645 m). This is the currently broadcast ellipsoid height, and it differs from the GPS-measured NAD 83 ellipsoid height (763.38 ft) by the geoid residual (-0.11 ft). This is done so the same geoid model, GEOID03, can be used at the rover to convert second ellipsoid heights back to NAVD 88 elevations. However, as we'll see later, this assumes that the geoid residual is the same everywhere, which we know it isn't. These geoid residuals have a mostly east-west tilt of about 2 parts per million (Figure 12).
hi	**0.00 ft**. The elevations h & H are already for the bottom of antenna mount.
ov	**0.17 ft** (0.053 m). This is the offset for the mechanical L1 phase center of the current Trimble Zephyr Geodetic antenna with ground plane (#41249-00), per Trimble antenna calibration. The variable portion is not shown here. Other non-Trimble antenna models, such as those produced by NGS or IGS, may use a different offset, computed from some other reference point.

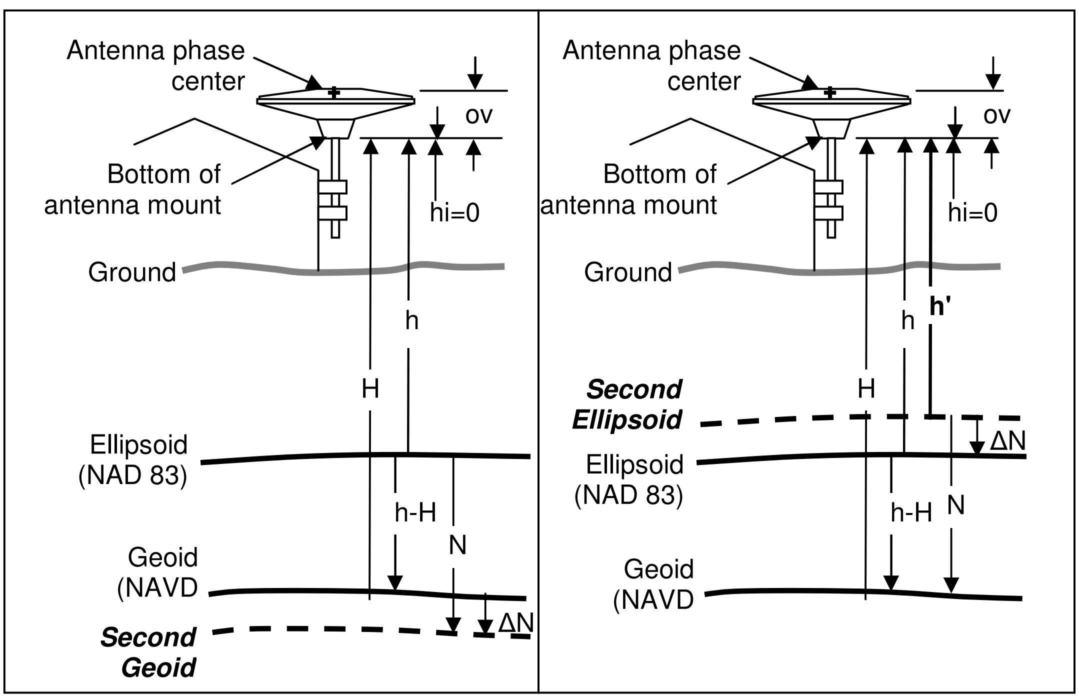


Figure 26: Geoid Residual Visualized as Second Geoid or Second Ellipsoid

4.3. SAMPLE CALUCATION TO MADISON S GPS (1.5 miles south): Table 21 and associated Figure 27 and Figure 28 show two sample elevation calculations from the Sayle Street Base to NGS Station MADISON S GPS. The only difference between Method 1 and Method 2 is in the geoid separations (N_1 & N_2). Method 1 uses geoid separations computed from published NAD 83 ellipsoid heights and NAVD 88 elevations at each station, and Method 2 uses the GEOID03 geoid model. The ellipsoid height component of the GPS baseline (Δh_{12}), the critical GPS-measured value, is not actually measured here. It is a 'perfect' baseline component computed from the published ellipsoid heights (h).

Using Method 1, the computed NAVD 88 elevation (H_2) at MADISON S GPS equals the NGS-published value. It matches because the calculation uses a geoid separation (N_2) computed from H_2, and the 'perfect' baseline. Using Method 2, the computed NAVD 88 elevation (H_2) at MADISON S GPS is 0.04 ft less than the NGS-published value. This is difference due entirely to using GEOID03 to estimate geoid separations, and equals the difference in the geoid residuals at the two stations (ΔN_1=-0.11 ft at Sayle Street Base Station, ΔN_2=-0.07 ft at MADISON S GPS).

Table 21: Sample Calculation, Sayle Street Base to MADISON S GPS

Method 1: Using NAD 83 Ellipsoid & "true" geoid separations	
H_1	875.69 ft *leveled NAVD 88*
+ N_1	**-(-112.31 ft) = *h_2-H_2***
= h_1	**763.38 ft** *NAD 83*
+ hi_1	+0.00 ft
+ ov_1	+0.17 ft *variation ignored*
+ Δh_{12}	-1.71 ft *inversed baseline component, computed from (h_2+hi_2+ov_2)-(h_1+hi_1+ov_1)*
– ov_2	-0.17 ft *assume same antenna*
– hi_2	-6.00 ft
= h_2	**= 755.67 ft** *NAD 83*
– N_2	**-(-112.24 ft) = *h_2-H_2***
= H_2	= 867.91 ft *leveled NAVD 88*

Method 2: Using Second Ellipsoid created from GEOID03 geoid separations	
H_1	875.69 ft *leveled NAVD 88*
+ N_1	**+(-112.42 ft) *GEOID03***
= h_1	**= 763.27 ft** *Second ellipsoid*
+ hi_1	+0.00 ft
+ ov_1	+0.17 ft *variation ignored*
+ Δh_{12}	-1.71 ft *same inversed baseline*
– ov_2	-0.17 ft *assume same antenna*
– hi_2	-6.00 ft
= h_2	**= 755.56 ft** *Second ellipsoid*
– N_2	**-(-112.31 ft) *GEOID03***
= H_2	= 867.87 ft

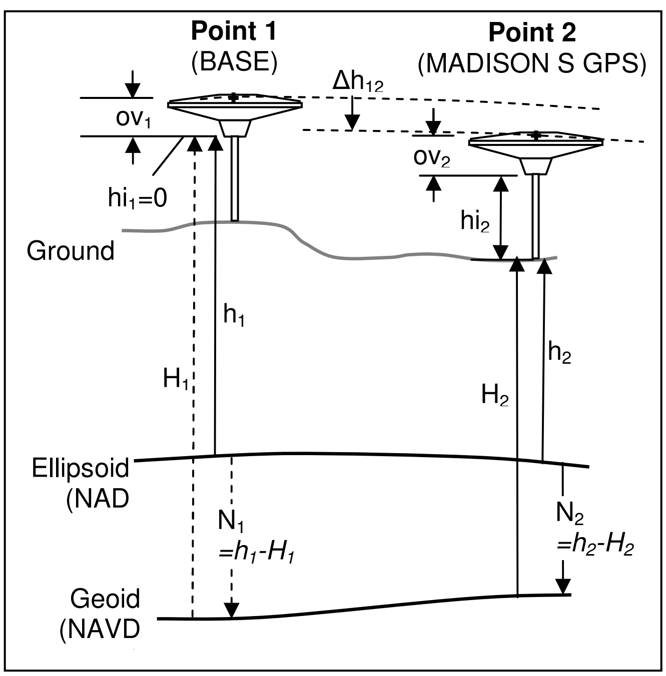


Figure 27: Method 1 Diagram

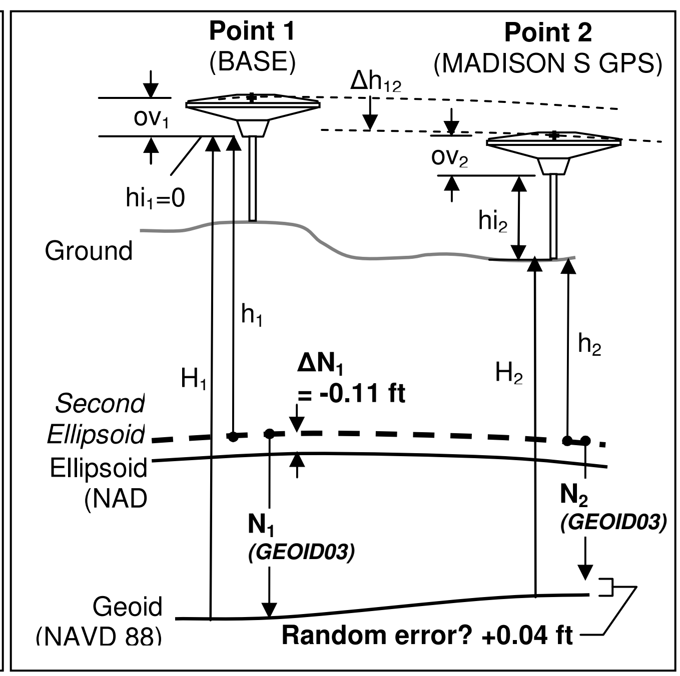


Figure 28: Method 2 Diagram

The difference in the final NAVD 88 elevation (H_2) of 0.04 ft using Method 2 isn't much, but MADISON S GPS is only 1.5 miles southerly of the Sayle Street Base, and we know from the GEOID03 shift & tilt analysis (Figure 13) that the GEOID03 north-south tilt is only 0.35 parts per million, which would add up to only 0.003 ft at a distance 1.5 miles south of the base. However, note that the best-fit shift in GEOID03 is -0.09 ft at the Sayle Street base (Figure 13), compared to the geoid residual of –0.11 feet at the Sayle Street Base (Table 20) and -0.07 feet at MADISON S GPS (Table 04 / Figure 10). The difference in the two geoid residuals is directly related to the 0.04 ft difference in the NAVD 88 elevations, but it cannot be concluded that Method 1 is correct and Method 2 is wrong. The inversed baseline (Δh_{12}) used in both Method 1 and 2 is computed from the GPS-measured NAD 83 ellipsoid heights (Method 1), either or both of which may have error. Less likely but still conceivable, GEOID03 may have error over this short distance, which isn't removed by shifting and tilting (i.e. a local gravitational variation). At any rate, a discrepancy of 0.04 feet is not statistically detectable with any but the most precise GPS techniques.

Method 3 (Table 22 / Figure 29) shows the same sample computation using the shifted and tilted GEOID03 instead of the unmodified GEOID03 (Method 2, Table 21 / Figure 28). The shift and tilt values determined from the 1997 NAD 83 ellipsoid and NAVD 88(1991) geoid are used (Figure 13). Since MADISON S GPS is so close to the Sayle Street Base Station, the shift and tilt values are the same (-0.09 ft) when rounded to 0.01 ft. Thus, the computed NAVD 88 elevation (H_2) is the same 0.04 ft smaller, as in Method 2.

Table 22: Sample Calculation, Sayle Street Base to MADISON S GPS

Method 3: Using NAD 83 Ellipsoid and Shifted & Tilted GEOID03	
H_1	875.69 ft *leveled NAVD 88*
+ N_1	**+(-112.33 ft)** *= -112.42 GEOID03 - (-0.09) shift & tilt*
= h_1	**763.36 ft**
+ hi_1	+0.00 ft
+ ov_1	+0.17 ft *variation ignored*
+ Δh_{12}	-1.71 ft *same inversed baseline*
– ov_2	-0.17 ft *assume same antenna*
– hi_2	-6.00 ft
= h_2	= **755.65** ft
– N_2	**-(-112.22 ft)** *= -112.31 GEOID03 - (-0.09) shift & tilt*
= H_2	= 867.87 ft

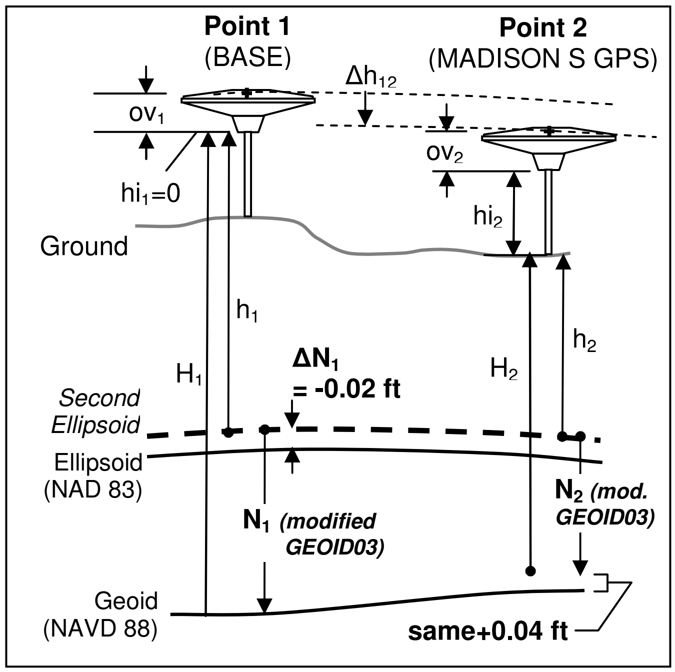


Figure 29: Method 3 Diagram

4.4. SAMPLE CALUCATION TO COTTAGE GROVE S GPS (9.3 miles east): Table 23, Figure 30 and Figure 31 show the same type of elevation calculations, from the Sayle Street Base to NGS Station COTTAGE GROVE S GPS, about 9.3 miles to the east. Note that Method 2's computed NAVD 88 elevation (H_2) is 0.14 feet less than Method 1's. This is significantly more than the 0.04-foot difference to MADISON S GPS (Table 21 / Figure 28), because COTTAGE GROVE S GPS is much farther away from the Sayle Street Base station and the 2.02 parts per million east-west tilt error in the unmodified GEOID03 geoid model is becoming significant ($2x10^{-6}$ x 9.3mi * 5280 ft/mi = 0.10 ft of error). The remaining 0.04 ft of the 0.14-ft discrepancy is due to random error in the published ellipsoid heights, NAVD 88 elevations, and/or the GEOID03 geoid model, as discussed in the previous example. The fact that the remaining difference is 0.04 ft in both examples is just coincidence.

Table 23: Sample Calculation, Sayle Street Base to COTTAGE GROVE S GPS

Method 1: Using NAD 83 Ellipsoid & "true" geoid separations	
H_1	875.69 ft *leveled NAVD 88*
+ N_1	**-(-112.31 ft) = *h_1-H_1***
= h_1	**763.38 ft** *NAD 83*
+ hi_1	+0.00 ft
+ ov_1	+0.17 ft *variation ignored*
+ Δh_{12}	+60.75 ft *inversed baseline component, computed from $(h_2+hi_2+ov_2)-(h_1+hi_1+ov_1)$*
– ov_2	-0.17 ft *assume same antenna*
– hi_2	-6.00 ft
= h_2	**= 818.13 ft** *NAD 83*
– N_2	**-(-112.60 ft) = *h_2-H_2***
= H_2	= 930.73 ft *leveled NAVD 88*

Method 2: Using Second Ellipsoid created from GEOID03 geoid separations	
H_1	875.69 ft *leveled NAVD 88*
+ N_1	**+(-112.42 ft) *GEOID03***
= h_1	**= 763.27 ft** *Second ellipsoid*
+ hi_1	+0.00 ft
+ ov_1	+0.17 ft *variation ignored*
+ Δh_{12}	+60.75 ft *same inversed baseline*
– ov_2	-0.17 ft *assume same antenna*
– hi_2	-6.00 ft
= h_2	**= 818.02 ft** *Second ellipsoid*
– N_2	**-(-112.57 ft) *GEOID03***
= H_2	= 930.59 ft

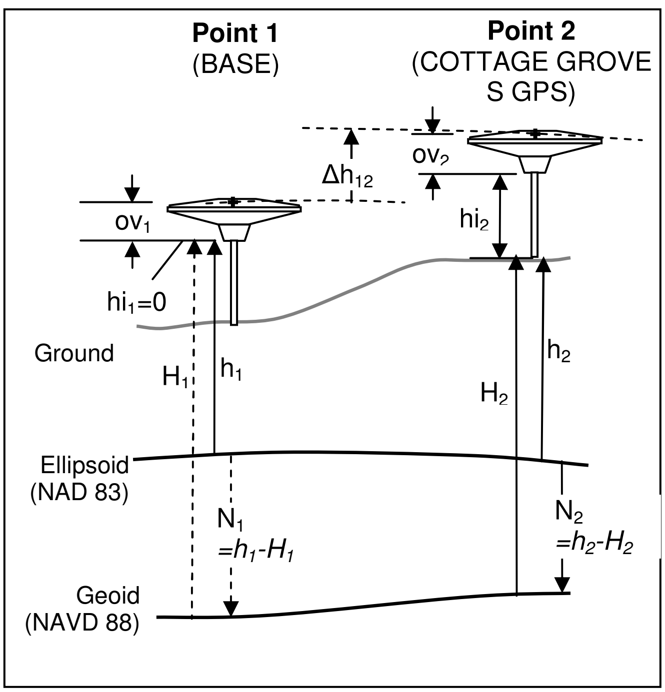


Figure 30: Method 1 Diagram

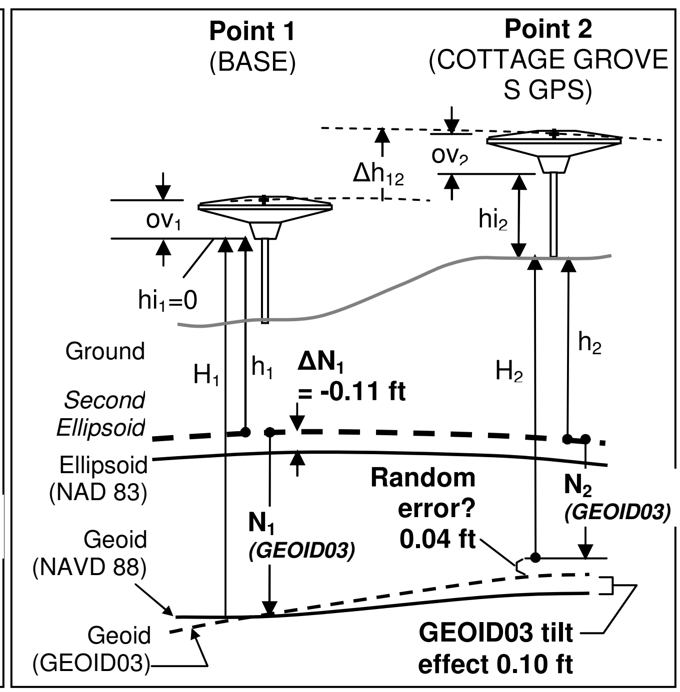


Figure 31: Method 2 Diagram

Method 3 (Table 24 / Figure 32) shows the same computation to COTTAGE GROVE S GPS using the shifted and tilted GEOID03. The 0.10 ft of geoid model tilt has been removed, leaving the 0.04 ft difference relative to Method 1. As with the MADISON S GPS example, the shift and tilt values were determined from the 1997 NAD 83 ellipsoid and NAVD 88(1991) geoid (Figure 13).

Table 24: Sample Calculation, Sayle Street Base to MADISON S GPS

Method 3: Using NAD 83 Ellipsoid and Shifted & Tilted GEOID03	
H_1	875.69 ft *leveled NAVD 88*
+ N_1	**+(-112.33 ft)** = *-112.42 GEOID03 - (-0.09) shift & tilt*
= h_1	**763.36 ft**
+ hi_1	+0.00 ft
+ ov_1	+0.17 ft *variation ignored*
+ Δh_{12}	+60.75 ft *same inversed baseline*
– ov_2	-0.17 ft *assume same antenna*
– hi_2	-6.00 ft
= h_2	**= 818.11 ft**
– N_2	**-(-112.58 ft)** = -112.57 *GEOID03 - (0.01) shift & tilt*
= H_2	= 930.69 ft

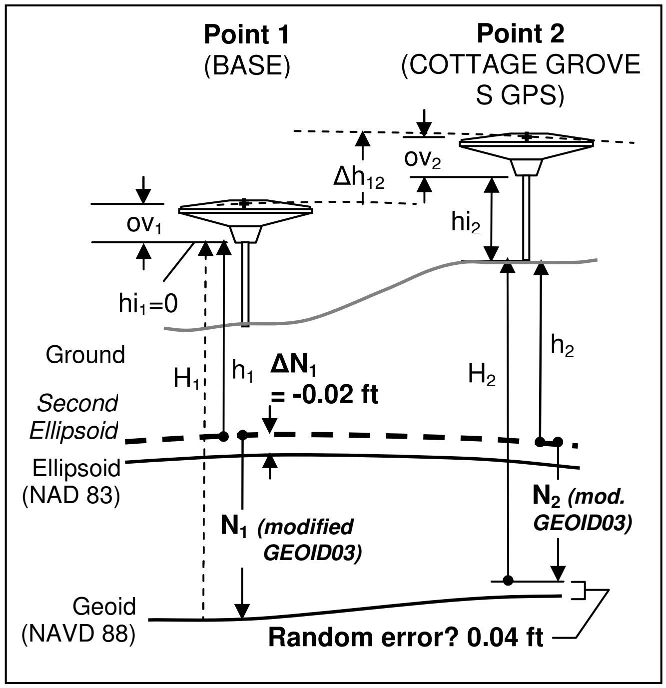


Figure 32: Method 3 Diagram

5.1. ACRONYMS USED:

Acronym	*Full Name*
ARP	Antenna Reference Point (for vertical positioning)
DOT	Department of Transportation
GEOID##	Geoid model produced by the NGS (e.g. GEOID03 in 2003, GEOID09 in 2009, GEOID12A in 2012)
GPS	Global Positioning System
NAD 83	North American Datum of 1983 (ellipsoid)
NAVD 88	North American Vertical Datum of 1988 (geoid)
NGS	National Geodetic Survey
NGVD 29	National Geodetic Vertical Datum of 1929 (geoid)
PID	Point Identifier (NGS Stations)
USGS	United States Geologic Survey
WISCORS	Wisconsin Continuously Operating Reference Stations (state-wide RTK GPS network)

5.2. ABOUT THE AUTHOR: Dan Rodman received a Master's Degree in Geospatial Information Engineering from the Department of Civil and Environmental Engineering at the UW-Madison, and is a Wisconsin Professional Land Surveyor. He has taught classes as a lecturer in surveying and GPS at the UW-Madison from 1999 to 2006 and worked as a surveyor in the both private sector and with the City of Madison.

APPENDIX 1: GEOID MODEL SHIFT & TILT COMPUTATION

Formula: $(1)*\Delta N_0 + (y-y_0)*\theta_y + (x-x_0)*\theta_x = \Delta N + v_{\Delta N}$

- x,y = horizontal position (x = easting, y = northing)
- x_0,y_0 = horizontal position of pivot point (origin)
- ΔN_0 = best-fit geoid shift at pivot point
- θ_y = tilt in y (northing) direction, computed in radians (effectively a unitless slope ratio)
- θ_x = tilt in x (easting) direction, computed in radians (effectively a unitless slope ratio)
- ΔN = observed "geoid residual" = GEOID03 minus difference in NGS heights = N-(h-H) *For example see Table 04 / Figure 10*
- $v_{\Delta N}$ = residual in observed "geoid residual" (once best-fit shift & tilt have been applied) *For example see Figure 13*

Least-squares matrix equations: **AX = L+V**

A matrix (parameters)	X matrix (unknowns)	L matrix (observations)	V matrix (residuals)
$\begin{bmatrix} 1 & y_1-y_0 & x_1-x_0 \\ 1 & y_2-y_0 & x_2-x_0 \\ 1 & y_3-y_0 & x_3-x_0 \\ 1 & y_4-y_0 & x_4-x_0 \\ \ldots & & \end{bmatrix}$	$\begin{bmatrix} \Delta N_0 \\ \theta_y \\ \theta_x \end{bmatrix}$	$\begin{bmatrix} \Delta N_1 \\ \Delta N_2 \\ \Delta N_3 \\ \Delta N_4 \\ \ldots \end{bmatrix}$	$\begin{bmatrix} v_{\Delta N 1} \\ v_{\Delta N 2} \\ v_{\Delta N 3} \\ v_{\Delta N 4} \\ \ldots \end{bmatrix}$

The signs of the resulting shift and tilt parameters indicate the direction in which the geoid model is off, rather than the corrections needed. This corresponds to Figure 09. For example (see Figure 12), a best-fit geoid residual (shift) (ΔN) of -0.09 ft at the old Sayle Street Base station means the unmodified GEOID03 geoid separation (-112.42 ft) at that point is -0.09 ft off the modified value. The modified value is obtained by subtracting the shift: -112.42 - (-0.09) = -112.33 ft. Use the sample values in Table 21 and Table 22 to verify that the signs are entered correctly. The origin shift calculations for Table 09 are also helpful for understanding.

The parameters defined above (x_0, y_0, ΔN_0, θ_y, θ_x) can be directly entered in the author's Trimble survey controller v12.46 software for a user-entered "geoid + inclined plane" vertical adjustment. Note that the origin and tilts are related to a northing, easting coordinate system, and thus require particular horizontal datum and map projection settings. See Table 09 and Table 10. Results should always be checked against independently-derived values, such as those provided in the separate document "Recommended RTK GPS Configuration Parameters for the City of Madison, WI Base Station."

Other GPS controllers may use different formulas. For example, Carlson's SurvCE v2.61 software (as of Sept. 2012) uses a formula with no offset origin (x_0,y_0 above):

Vertical Adjustment = $(-a*x - b*y - d)/c$, which can be rearranged as:
Vertical Adjustment = $(-a/c)*x + (-b/c)*y + (-d/c)$

Where:

- Vertical Adjustment = ΔN = geoid residual
- x,y = horizontal position (x = easting, y = northing)
- $-b/c = \theta_y$ = tilt in y (northing) direction, computed in radians (effectively a unitless slope ratio)
- $-a/c = \theta_x$ = tilt in x (easting) direction, computed in radians (effectively a unitless slope ratio)
- $-d/c = \Delta N_0$ = best-fit geoid shift at coordinate origin (x,y=0,0) *See Table 09 example for comparing origins*

Carlson SurvCE v2.61 allows a vertical calibration to be computed by sets of user-entered geodetic and local coordinates. Note that if incorporating a geoid model, the geodetic "elevation" is the ellipsoid height minus the unadjusted geoid separation (h – N in Figure 09). The software adds the "Vertical Adjustment" (ΔN) to the "elevation" (h – N) to get the adjusted local orthometric elevation ($h – N + \Delta N = H$). This matches the formula $\Delta N = N – (h-H)$ in Figure 09.

The products mentioned above are included only as examples to enhance understanding, not as endorsements.

APPENDIX 2: OLD SAYLE STREET BASE STATION ELEVATION HISTORY

Note: additional information is available on the web at http://gis.cityofmadison.com/Madison_GPS/

Circa-2005 – NAVD 88 Elevation: According to City Engineering records, Ayres & Associates performed 3-wire leveling from NGS Station 2V02 (PID #DF9800) to a survey nail in asphalt under the base station. The nail was determined to be 0.30 ft below 2V02, and a tape measure was used to determine the bottom of antenna mount as 21.64 ft above the nail. This put the bottom of antenna mount (-0.30 + 21.64) = 21.34 ft above 2V02. Using the NAVD 88(1991) NGS-published elevation (dated 2004) for 2V02 of 854.35ft (260.406m), the bottom of antenna mount was computed as (854.35 – 0.30 + 21.64) = 875.69 ft, NAVD 88(1991). The nail was missing as of 2007.

Circa-2005 – NAD 83 Ellipsoid Height: The 1997 NAD 83 ellipsoid height of the bottom of antenna mount was determined as 763.38 ft (232.679 m) by a University of Wisconsin Civil & Environmental Engineering Department project, based on a network of static GPS baselines fixed to the 1999 NAD 83 ellipsoid heights of three NGS stations: COTTAGE GROVE GPS = 788.30 ft (240.274 m), ROCK = 946.57 ft (288.515 m), and KOLLATH = 1027.99 ft (313.331 m). 95% confidence of the adjusted base station height was +/- 0.01 m, although discrepancies of up to 4 cm relative to NGS published NAD 83(97) ellipsoid heights were found at other stations in the network.

The UW-determined ellipsoid height was confirmed by the NGS Online Positioning User Service (OPUS). An OPUS ellipsoid height for the bottom of antenna mount was determined as 763.36 ft (232.673 m) by a weighted average of fifteen 24-hour observations randomly selected between September 2004 and February 2005. The largest "peak-to-peak" error from the OPUS solutions (the range of height values solved from the different base stations) was 0.15 ft (0.045 m). The OPUS datum was NAD 83(CORS96)(EPOCH 2002.0000), which is considered to be very close to NAD 83(1997). Base stations BLRW, LCDT and MIL1 were used. Ellipsoid heights of the antenna reference points were: LCDT = 624.92 ft (190.477 m) as of 11/2001; BLRW = 628.97 ft (191.711 m) as of 03/2002; and MIL1 = 487.00 ft (148.437 m) as of 03/2002.

Other ellipsoid heights have been established for the base station, but they are computed from published NAVD 88 elevations and NGS geoid models, not directly measured from published NAD 83 ellipsoid heights. For example, beginning in 2005, the city-published ellipsoid height for the bottom of antenna mount was 763.27 ft (232.645 m), computed from the leveled NAVD 88 elevation of 875.69 ft (266.911 m) and the GEOID03 geoid separation -112.42 ft (-34.266 m). Note that the Trimble base station software's Real-Time Kinematic (RTK) correction signal broadcasts the ellipsoid height of the mechanical L1 phase center, which, using Trimble's antenna calibration parameters, is 0.17 ft (0.053 m) above the bottom of antenna mount for the current Zephyr Geodetic antenna (model 41249.00), giving a broadcast ellipsoid height of 763.44 ft (232.698 m).

Circa-2005 – Geoid Separation: The geoid separation (N) predicted by the NGS's GEOID03 geoid model at the base station is -112.42 ft. In contrast, the difference between the UW's GPS-measured 1997 NAD 83 ellipsoid height and Ayres & Associates leveled NAVD 88(1991) elevation was (h – H) = (763.38 ft - 875.69 ft) = -112.31 ft. The GEOID03 geoid separation is –0.11 ft different (more negative, meaning larger in magnitude) from the difference in ellipoid height and elevation. This difference is within 0.02 ft of the –0.09 ft shift computed at the base station for shifting and tilting GEOID03 to best-fit published 1997 NAD 83 ellipsoid heights and NAVD 88(1991) elevations at nearby NGS stations (Figure 12 & Figure 13). This comparison is independent confirmation that the ellipsoid heights and NAVD 88 elevations measured by the UW and Ayres in 2005 were in good agreement with their respective datums.

Nov. 2007 – NAVD 88 Elevation: Dan Rodman, City Parks Division Surveyor, performed 3-wire leveling on Nov 16, 2007 from NGS Station 2V02 (PID #DF9800) to a new survey nail set in epoxy under the base station. The old nail set by Ayres was missing. The new nail was determined to be 0.324 ft below 2V02 (level loop misclosure 0.003 ft), and two different steel tapes were used to determine the bottom of antenna mount as 21.646 ft (discrepancy 0.005 ft) above the new nail. This put the bottom of antenna mount (-0.324 + 21.646) = 21.32 ft above 2V02. Using the NAVD 88(2007) NGS-published elevation for 2V02 of 854.44 ft (260.433 m), the bottom of antenna mount was computed as (854.44 – 0.324 + 21.646) = 875.76 ft (NAVD 88(2007)).

As a check, an additional offset survey nail was set in asphalt 36.0 ft east-southeast of the base station. That nail was determined to be -0.891 ft below 2V02 (same level loop as above), and a total station with reflectorless distance measurement was used to determine the bottom of antenna mount as 22.20 ft above the new nail. This put the bottom of antenna mount (-0.891 + 22.20) = 21.31 ft above 2V02. Using the NAVD 88(2007) NGS-published elevation for 2V02 of 854.44 ft (260.433 m), the bottom of antenna mount was computed as (854.44 – 0.891+ 22.20) = 875.75 ft (NAVD 88(2007)), verifying the steel-taped vertical distance within 0.01 ft.

Comparing 2007 leveling to 2005 leveling, the elevation change from 2V02 to the bottom of antenna mount was (-0.30 + 21.64) = +21.34 ft in 2005, compared to (-0.324 + 21.646) = +21.32 ft in 2007. This difference appears to be larger than measurement error given the precise 3-wire leveling technique used, and could suggest settling over time at the base station location. For that matter, at station 2V02, the NAVD 88(2007) elevation is 854.44 ft, which is +0.09 ft larger than its NAVD 88(1991) elevation (dated 2004) of 854.35 ft. The difference of +0.09 ft is 0.02 ft smaller than the average 1991 to 2007 NAVD 88 shift of 0.11 ft (Figure 02), which suggests the possibility of settling at 2V02. The NGS lists 2V02 as "Stability B" (probably hold elevation and position well), but it is on a bridge on John Nolen Drive, most of which is built on fill. Of course, this would imply that the entire Madison isthmus had settled 0.03-0.04 ft within the recent past (Figure 03).

Assuming 2V02 settled anywhere from 0.00 - 0.02 ft relative to datum, and the base settled 0.02 ft relative to 2V02, the base station would have settled 0.02 - 0.04 ft relative to datum from 2005 to 2007. However, this difference could also plausibly be due to measurement error rather than movement. More rigorous leveling to multiple bench marks would be required to determine this. Regardless, a difference of this magnitude is not reliably detectable with any but the most precise GPS techniques.

2007 – NAD 83 Ellipsoid Height: To date (November 2007), no new GPS survey has been performed to tie the base station to local NGS stations to recompute the ellipsoid height.

The NGS Online Positioning User Service (OPUS) was used to redetermine the ellipsoid height of the base of antenna mount. Nineteen 24-hour sets data between Sept. 26 and Nov. 3, 2007 were processed to produce a weighted-average ellipsoid height of 763.29 ft (232.652 m). Weights were computed using the inverse of the square of the OPUS "overall RMS" value. The precise ephemeris was used. Base stations used were BLRW, LCDT and WIM5. BLRW and LCDT are the same 2 base stations used for the 2005 OPUS results, but the third, MIL1, is no longer operational. For comparison, the same data processed with base stations BLRW, LCDT and RIS5 (instead of WIM5) produced a weighted average ellipsoid height 0.01 ft lower. The range of the nineteen 24-hour solutions (using BLRW, LCDT & WIM5) was from 763.27 ft (232.645 m) to 763.33 ft (232.662 m), with a standard deviation of 0.02 ft (0.005 m). The largest peak-to-peak error (discrepancy among base stations) for any day's data was 0.24 ft (0.073 m). The OPUS datum is still NAD 83(CORS96)(EPOCH 2002.00), but this is supposed to be essentially identical to NAD 83(NSRS2007).

The 2007 OPUS ellipsoid height of 763.29 ft, is 0.09 ft lower than the 2005 OPUS value of 763.38 ft, and 0.07 ft lower than the UW-determined 2005 NAD 83 ellipsoid height of 763.36. This difference should not be a result of datum shift since 1997 NAD 83 ellipsoid heights are about the same as 2007 ellipsoid heights in the base station area (Figure 08), and the OPUS datum, NAD 83(CORS96)(Epoch 2002), is essentially the same as NAD 83(2007). However, the OPUS solutions involve GPS baselines up to 90 miles long, which are conceivably subject to height errors of this magnitude, even after averaging. It is difficult to say that the base station has in fact settled from the OPUS data alone.

2007 – Geoid Separation: The geoid separation (N) predicted by the NGS's GEOID03 geoid model at the base station is -112.42 ft. In contrast, the difference between the 2007 ellipsoid height from OPUS (considered to be in the NAD 83(NSRS2007) datum) and The leveled NAVD 88(2007) elevation was (h – H) = (763.29 ft - 875.76 ft) = -112.47 ft. The GEOID03 geoid separation, -112.42 ft, is +0.05 ft different (less negative, meaning smaller in magnitude) from the difference in ellipoid height and elevation, -112.47 ft. This +0.05 ft difference is within 0.02 ft of the +0.03 ft shift computed at the base station for shifting and tilting GEOID03 to best-fit published 2007 NAD 83 ellipsoid heights and NAVD 88(2007) elevations at nearby NGS stations (Figure 15). This basically means the OPUS-derived ellipsoid height and the leveled NAVD 88 elevation agree to within 0.02 ft.
Note that the 2005 GPS-measured ellipsoid height and 2005 leveled NAVD 88 elevation also agreed to 0.02 ft, but in the opposite direction. This would indicate a 0.04 ft drop in the base station vertical position from 2005 to 2007, but again, this difference is conceivably within the error margin of the measurements.

Nov. 2008 – NAD 83 Ellipsoid Height: See Nov. 2008 ellipsoid height for Emil Street base. Sayle Street base was determined as 232.665 m (763.34 ft) (bottom of antenna mount = Antenna Reference Point), relative to the ellipsoid height of 230.327 m adjusted April 2007 for MADISON S GPS (NGS PID# DF9799). 232.665 m is 2 cm more than the current (Dec. 2008) broadcast ellipsoid height of 232.645 m, which is not a true NAD 83 ellipsoid height and therefore not directly comparable. The 232.665 m ellipsoid height compares well with 2005 OPUS value (232.673 m; +8 mm) and the circa-2005 UW survey (232.679m; +14 mm). As with the 2007 leveling and OPUS data, this new ellipsoid height does indicate a possible drop of around 1 cm (0.03 ft) of the Sayle Street base station relative to datum.

Jan 2009 – NAD 83 Ellipsoid Height: WDOT detemined an ellipsoid height for the Sayle Street base ARP = 232.672 m (763.36 ft), relative to the Emil St base ARP ellipsoid height of 248.694 m determined by WDOT for WISCORS. See Jan 2009 note for Emil St base in Appendix 3. WDOT used 44 hours of data over about 50 days between the Emil St and Sayle St base. Datum is NAD 83(2007). This height is 7 mm larger than Rodman's Nov

2008 ellipsoid height, and the WDOT's relative difference from Emil to Sayle is only 3 mm different than Rodman's Nov. 2008 data. The WDOT's ellipsoid height of 232.762 m compares well with the ~2005 OPUS (232.673 m) and UW (232.679 m) values as well.

Using the adjusted NAVD 88(1991) – 2007 NAD 83 geoid separation (Table 08) at Sayle St of –34.266 – (–0.025) = –34.241 m, the NAVD 88(1991) elevation would be 232.672 – (-34.241) = 266.913 m (875.70 ft), which is only 0.01 ft higher than the NAVD 88(1991) elevation of 875.69 ft determined ~2005 by Ayres relative to the same station 2V02.

Using the adjusted NAVD 88(2007) – 2007 NAD 83 geoid separation (Table 08) at Sayle St of –34.266 – (+0.010) = –34.276 m, the computed NAVD 88(2007) elevation would be 232.672 – (-34.276) = 266.948 m (875.81 ft), which is 0.05 ft higher than the NAVD 88(2007) elevation of 875.76 ft determined by Rodman in Nov. 2007 from NGS station 2V02. The slight differences in ellipsoid heights and elevations at Sayle St from 2005 to 2009 do not present conclusive evidence of actual movement.

APPENDIX 3: EMIL STREET BASE STATION ELEVATION HISTORY

Note: The abbreviation MAON ARP refers to the Emil Street base station (named MAON in WISCORS), at the Antenna Reference Point (ARP), which is the bottom of antenna mount (bottom of antenna housing).

Nov. 2008 – NAVD 88 Elevation: Dan Rodman (City Parks Div) performed 3-wire leveling from MADISON S GPS (NGS PID# DF9799) with a micrometer, invar rod & balanced sight distances (loop closure 3 mm), and a network of taped heights and direct-reverse averaged trigonometric elevations to MAON ARP (residuals all < 2 mm, except one check between bench marks = 5 mm). Estimated 95% confidence relative to MADISON S GPS is +/- 6 mm at ground station EMIL GPS (NGS PID# DG4231) and +/- 8 mm at MAON ARP (bottom of antenna mount).

Adjusted NAVD 88(2007) elevations relative to MADISON S GPS (April 2007 elev = 264.569 m) were:
MAON ARP = 282.925 m (928.23 ft) (bottom of antenna mount = Antenna Reference Point)
EMIL GPS = 271.767 m (891.62 ft)

The NAVD 88(1991) elevation at MADISON S GPS is 264.538 m, which is 0.031 m (0.10 ft) less than the NAVD 88(2007) elevation of 264.569 m. That's a typical shift in Madison, where most are +0.09 to +0.13 ft. Because MAON ARP is only ~1 mile west of MADISON S GPS, the NAVD 88(1991) elevation is estimated to be 0.031 m (0.10 ft) less.

Nov. 2008 - NAD 83 Ellipsoid Height: Dan Rodman (City Parks Div) adjusted 4 days (~96 hours) of data between the old Sayle Street base & MAON ARP, together with three 30-minute sessions at MADISON S GPS at different times on different days. 95% confidence from adjustment at both bases was +/- 6 mm vertical relative to MADISON S GPS.

Adjusted 2007 NAD 83 ellipsoid heights were relative to MADISON S GPS (2007 ellip ht = 230.327 m) were:
Sayle Base ARP = 232.665 m (763.34 ft) (bottom of antenna mount = Antenna Reference Point)
MAON ARP = 248.690 m (815.91 ft) (bottom of antenna mount = Antenna Reference Point)

The MAON ARP ellipsoid height was verified by OPUS: one 24-hour data set from Nov 28, 2008 processed relative to CORS base stations BLRW (PID# AI2149), RIS5 (PID# DI8416) and WIM5 (PID# DI2110) gave an ellipsoid height for MAON ARP of 248.691 m (only 1 mm different), with a peak-to-peak error (range of 3 baseline solutions) of 0.031 m. OPUS datum is NAD_83(CORS96)(2002), which has been found to be essentially the same as NAD 83(2007) in the area. As of Jan. 2009, Problems with Trimble data produced by the NetR5 receiver at MAON ARP have prevented further processing with OPUS or OPUS-RS.

Jan. 2009 – NAD 83 WISCORS Ellipsoid Height: The Wisconsin DOT determined an ellipsoid height for MAON ARP of 248.694 m (815.92 ft), This height was determined using 44 hours of data over about 50 days relative to adjacent WISCORS and CORS stations, which have positions in the NAD 83(CORS96)(2002) datum, which is essentially identical to NAD 83(2007) in Wisconsin. The WISCORS ellipsoid height is only 4 mm (0.01 ft) larger than Rodman's Nov. 2008 ellipsoid height determined from local NGS station MADISON S GPS, which shows good vertical agreement between the local height modernization station and the statewide CORS network.

Nov. 2008 – Geoid Separation: The geoid separation at MAON ARP computed from the NAVD 88(2007) – 2007 NAD 83 best-fit geoid shift and tilt values in Table 09 is -34.237 m (-112.33 ft). That is, -34.229 m per GEOID03, minus shift of +0.008 m (no tilt factors since computed at the origin / pivot point).

The difference between Rodman's Nov. 2008 surveyed 2007 ellipsoid height and 2007 elevation for MAON ARP is (248.690 - 282.925) = -34.235 m, which is only 0.002 m (0.007 ft) different. The difference between the WISCORS

Jan .2009 ellipsoid height and Rodman's Nov. 2008 elevation is (248.694 – 282.925) = -34.231 m, which is 0.006 m (0.02 ft) different.

This indicates good agreement between the leveled elevation and GPS-measured ellipsoid height relative to MADISON S GPS. The discrepancies are well within RTK accuracy and the fit of the geoid model shift and tilt computation.

Sept. 2012 – Local Elevation Check: Dan Rodman (City Parks Div.) checked elevation of MAON ARP from the local bench marks set in Nov. 2008. The local measurements put the antenna 0.00 to 0.01 ft lower than before, but 95% confidence is estimated at a few hundredths of a foot (based on total station direct/reverse trigonometric elevations).

Oct-Nov. 2012 - NAD 83(2011) WISCORS Ellipsoid Height: The Wisconsin DOT determined an ellipsoid height for MAON ARP of 248.669 m (815.84 ft) in the NAD 83(2011)(EPOCH 2010.00) datum, relative to National CORS stations in and around Wisconsin, using 13 days of 24hr observation files during Oct-Dec 2012. That ellipsoid height is 0.025 m = 0.08 ft smaller than the DOT's previous 2007 NAD 83 ellipsoid height. In comparison, NGS station ellipsoid heights are 0.08 to 0.12 ft smaller (average 0.10 ft) in the Madison area, 0.09 & 0.10 ft smaller at stations nearest MAON (Figure 08 & Table 03).

2012: GEOID12A Geoid Separation: MADISON S GPS was destroyed in 2010 but included in the NAVD 88(2012) adjustment. The Nov. 2008 leveled elevation difference from MADISON S GPS to MAON ARP of (282.925 – 264.569 m) = +18.356 m = +60.22 ft, added to the NAVD 88(2012) elevation of 264.588 m at MADISON S GPS, gives a NAVD 88(2012) elevation at MAON ARP of 282.944 m = 928.29 ft. Adding the GEOID12A geoid separation of -34.278 m at MAON gives a NAD 83(2011) ellipsoid height of 248.666 m for MAON ARP, 0.003 m (0.01 ft) lower than the DOT's Oct-Nov 2012 ellipsoid height of 248.669 m. GEOID12A is designed to convert NAD 83(2011) ellipsoid heights to NAVD 88(2012) elevations, and the agreement between these different measurements is very good.

Jan-Feb 2013 – NAD 83(2011) Ellipsoid Height Dan Rodman (City Parks Div.) performed static GPS on Jan 15 & Feb 13 2013 from nearby NGS stations MIDDLETON GPS (published ellip ht 294.943 m) and FITCHBURG N GPS (published ellip ht 251.186 m). From each station, the Feb. 13 observations differed by 0.01 ft or less from the Jan. 15 observations. The following NAD 83(2011) ellipsoid heights were computed for MAON ARP:

from FITCHBURG N GPS = 248.679 m (0.009 m = 0.03 ft higher than DOT's Oct-Nov 2012)
from MIDDLETON GPS = 248.658 m (0.012m = 0.04 ft lower than DOT's Oct-Nov 2012)

The observed ellipsoid height difference from MIDDLETON GPS to MAON ARP is within 0.01 ft of the same difference determined with static GPS on Dec 12 2008.